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OF CRUDE OIL:
AN ECONOMETRIC MODEL

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AN ECONOMETRIC MODEL

by

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An examination of the responsiveness to economic incentives of the U.S. petroleum industry is vital if the nation's oil supply is to be maintained or possibly increased. The identification of crucial variables that regulate this flow can provide insight on the impact of alternative petroleum policies on the generation of reserves and ultimately the prices and availability of crude oil products to the consumer.

The objective of this paper is to examine the responsiveness to price incentives of petroleum exploration, the generation of proven reserves and the production out of reserves in the United States.¹ First, a theoretical framework of oil extraction and supply is developed. Next, an econometric model, consisting of 11 stochastic equations and 3 identities, is constructed to capture the decisions affecting the supply of new discoveries, the increase of proven reserves and the production out of reserves.² The parameters of the structural relationships are simultaneously determined and are estimated by three-stage least squares. Finally, the sample period performance and predictability of the model are evaluated.

The Discovery and Extraction of Crude Oil

The domestic crude oil producing industry is subdivided into three stages: exploration, development, and extraction. Exploration consists of the geological and geophysical analyses which locate potential oil and/or natural gas deposits, and the drilling of exploratory wells. Once oil or gas have been discovered by exploration, development wells are drilled to estimate "proven" reserves. Finally, extraction

from existing wells occurs through the utilization of a number of different recovery methods.³ It can thus be said that petroleum exploration and development create new reserves. The amount of available reserves in turn influences the decisions concerning extracted oil to be refined in petroleum products. In what follows, close attention will be paid to the decision making behavior affecting all industry stages beginning with the flow of oil to the refineries and working upstream to the exploration stage.

Total Refinery Inputs

Crude oil is used in the refinery process, along with natural gas liquids, for conversion into a number of refined products, each suited to particular uses. Therefore, the demand for oil is derived from the aggregate of demands for refined products. The primary inputs to the U.S. petroleum refining industry are domestic crude oil, lease condensate, natural gas liquids and oil imports. In the past, domestic supplies constituted the bulk of primary inputs, but in recent years the contribution of foreign oil has grown rapidly. The various inputs into the refinery process at time period t can be summarized by the following identity:

$$\text{DISTR}_t = S_t + M_t + \text{NG}_t + \text{GA}_t \quad (1)$$

where the supply of crude oil and lease condensate (S_t), oil imports (M_t), the amount of natural gas liquids (NG_t) added for the refining processes, and the processing gain (GA_t) realized in the refineries should sum up to the total amount of refined products produced (DISTR_t)

during a particular year. Natural gas liquids and the processing gain have been a relatively small proportion of total inputs, averaging about 14% and 3% of total supply respectively over the 1959-1972 period.⁴

Production Out of Reserves

Given the demand for refined products, decisions must be made concerning the rate of crude oil production out of reserves. It must be noted that most costs of oil production are for exploration and development, and are incurred before production begins. The supply of domestic production is determined by the marginal cost of developing existing reserves because of the desire to balance annual flows with reserve levels. Marginal production costs will, therefore, depend on reserve levels relative to domestic production, and as the reserve to production ratio declines, we would expect marginal costs to rise sharply.⁵ This is because the greater the output, the more oil must be drawn from high cost, distant, low-quality sources. The average fixed cost per million barrels falls until all oil is extracted. The lower is the demand, the more output will be concentrated in the low-cost fields; and the greater the demand, the more oil will be called out from the higher cost supplies in response to actual or expected price increases.

Development of new production sites will be undertaken only if the expected price of oil covers all costs including a normal return on investment and a risk allowance.⁶ Complex interrelationships exist

between oil prices, the cost of production, investment decisions and production out of reserves. A short-term glut may send current prices down. If this was expected to be a temporary condition, and long-run price expectations were high enough, development might continue even at low current oil prices.⁷ Thus, the production rate out of reserves responds to expected rather than current prices.

Price expectations are represented in the model by a distributed lag of past prices (EP) and the opportunity cost of investing in petroleum production is expressed as a distributed lag of the difference between net price and the rate of interest (EZ).⁸ The marginal production (development) cost (MCO) is an exponential function of the supply of production (S), reserves at the beginning of the year (TR) and the opportunity cost of extracting the exhaustible stock resource (EZ):⁹

$$MCO_t = h_0 e^{(h_1 S_t - h_2 TR_t - h_3 EZ_t)} \quad (2)$$

Setting expected prices equal to marginal development costs and taking the logarithms of both sides, the equation to be estimated becomes:

$$S_t = c_0 + c_1 \ln EP_t + c_w TR_t + c_3 EZ_t \quad (3)$$

Imports, Natural Gas Liquids, Processing Gain and Price

Imports of crude petroleum are assumed to respond to domestic economic policy as well as the price of imported crude oil [5]. The price of imports can be treated as exogenous to the model since it is determined in the world petroleum market. Current imports (M), viewed as demand for foreign crude oil, are a function of imported oil in the previous time period, the price of imports (PM), the domestic supply of

crude (s) since import quotas were set on the basis of domestic output, and the utilization of domestic refining capacity (REF) which acts as a capacity constraint:

$$M_t = f(M_{t-1}, PM_t, S_t, REF_t) \quad (4)$$

A negative relationship between imports, import price and domestic refinery capacity utilization is expected.

The refinery process utilizes not only crude oil and lease condensate but also natural gas liquids. The amount of natural gas liquids added, because of economic and technological factors, has been steadily increasing over time. The quantity of natural gas liquids (NG) is positively related to the price of crude relative to the price of natural gas liquids (P/PNG) and a time trend (T^2):

$$NG_t = f(P_t/PNG_t, T^2) \quad (5)$$

The processing gain represents the expansion of fuels owing to some refining processes such as reforming and cracking, and is the final component needed to determine the total amount of refined liquids. The quantity of processing gain (GA) increases in direct proportion to the amount of crude oil and lease condensate run through stills (S), and declines in proportion to the amount of natural gas liquids added for refining (NG). A nonlinear time trend (T^2) has been added to the estimating equation:

$$GA_t = f(NG_t, S_t, T^2) \quad (6)$$

The price of crude oil (P) is assumed to be positively related to the sales of refined products and the price of crude imports (PM),

and negatively related to the price of natural gas (PNG) and the extent of refinery capacity utilization (REF). A distributed lag of the sales of refined products (DDISTR) rather than actual sales are used, because a sustained increase in sales of refined products must occur if the price of crude petroleum is to increase. Hence, the price equation of the model is:

$$P_t = f(\text{PNG}_t, \text{PM}_t, \text{REF}_t, \text{DDISTR}_t) \quad (7)$$

Supply of New Reserves

The decisions concerning crude oil production at the well-head are influenced in large part by the amount of oil reserves available in any time period. Additional reserves come from new discoveries, previous discovery extensions and revised estimates of previous discoveries. Thus for any time period t , total proven reserves of crude oil (R) are given by the identity:

$$R_t = R_{t-1} + \text{DC}_t + \text{EC}_t + \text{RC}_t - S_t \quad (8)$$

where extensions (EC), revisions (RC) and new discoveries (DC) during the year are combined to form additions to reserves. The amounts of crude oil extracted (S) are the only major subtraction from reserves.

Crude oil reserve extensions (EC) vary positively with expected prices (EP) and inversely to the amounts of crude previously discovered through exploratory drilling. Economic incentives account for the use of either new technologies or making present secondary and tertiary recovery methods economical. Furthermore, if discoveries (DC) at any

point in time are small, an incentive exists for the recovery of oil from already existing reservoirs by recovery from greater depths.

The equation to be estimated becomes:

$$EC_t = f(DC_{t-1}, DC_{t-2}, EP_t) \quad (9)$$

where EP represents expected prices expressed as a distributed lag of the crude oil price.

Apparently, revisions of established reserve levels (RC) do not respond to any specific economic or technological variables [16, p. 78]. Therefore, revisions are assumed to be proportional to changes in reserve levels (ΔR_{t-1}):

$$RC_t = f(\Delta R_{t-1}) \quad (10)$$

The supply of additional proven reserves (DC) is generated by determining the number of exploratory wells drilled (TED) and the average discovery size per well (ADSZ). The amount of new discoveries is then the product of discovery size and wells drilled:

$$DC_t = ADSZ_t \times TED_t \quad (11)$$

In geophysical exploration, as the major structures (oil pools) are discovered and tested, the search must increasingly turn to more subtle structural features (in terms of difficulty of drilling, thickness of the productive stratum and permeability of the formation). Generally, deposits occurring in such features are less prolific producers than previously found fields in more favorable structures.

Even if adequate incentive exists to encourage an intensified exploration effort, physical capacity limits the amount of exploration that can be accomplished within a given period of time. That limit is determined largely by the number of available drilling rigs and the

rate at which the drilling can be done. Considerable progress has been made in increasing drilling speed and lowering drilling costs. Further improvements not only will speed up the rate of exploration and site development but will also make some sites that previously did not warrant development, economical.

It follows that price incentives not only influence the amount of exploration, but also determine exploration characteristics. Higher incentives, for example, lead to more wildcat drilling on poorer prospects. Thus, the decision concerning the number of new exploratory wells drilled (TED) depends upon the inter-relationships between expected returns, costs of production and risk.

Following the work of Lintner [14], Mossin [17], Sharpe [21], and MacAvoy and Pindyck [16] we assume that risk can be represented by the variance of expected net dollar receipts. Hence, the present value in *certainty equivalent terms* of the net cash flow to the j th firm (V_j) is given by:

$$V_j = (1/r) (\bar{\pi}_j - \lambda \sigma_j) \quad (12)$$

where:

$\bar{\pi}_j = E(\tilde{\pi}_j)$, the expected net return cash flow to firm j

σ_j = the variance of $\tilde{\pi}_j$

λ = an index of risk aversion

r = a long-term market rate of interest.

Each firm will make drilling decisions which maximize V_j . Since wells may supply either oil and/or natural gas, the expected net return $E(\tilde{\pi}_j)$ from drilling a number of wells by the j th firm (TED_j) can be expressed as:

$$E(\tilde{\pi}_j) = TED_j [k(P_e \cdot ADSZ_j + PNG_e \cdot SZNG_j) - (C_d + C_x)] \quad (13)$$

where:

k = a multiplicative constant accounting for discoveries that may be extended or revised at a later date.

P_e , PNG_e = the expected prices of crude oil and natural gas respectively

$ADSZ$, $SZNG$ = the average discovery size of crude oil and natural gas respectively, and

C_d , C_x = the expected costs of development and exploration respectively.

Total exploration costs and subsequent total development costs can be expressed, following MacAvoy and Pindyck [16], as a function of the number of wells drilled (TED_j) and the average cost per well (ACW_j) for the j th firm. The equation which holds for all firms and maximizes net cash flow is:

$$TED = \gamma_0 + \gamma_1 (P_e ADSZ + PNG_e SZNG) + \gamma_2 R + \gamma_3 ACW \quad (14)$$

where R is the variance of dollar receipts over all the exploratory wells drilled.¹⁰ Because of the time-lag between investment outlays and the accrual of revenues, the interest rate (INT) is included as an additional variable. The estimating equation for the number of new exploratory wells drilled at time t by all firms becomes:

$$TED_t = f\{ACW_t, INT_t, [4\sigma_0^2 ADSZ_t^2 (P_{t-1} + P_{t-2} + P_{t-3})^2/9 + 4\sigma_G^2 SZNG_t^2 (PNG_{t-1} + PNG_{t-2} + PNG_{t-3})^2/9], [ADSZ_t (P_{t-1} + P_{t-2} + P_{t-3})/3 + SZNG_t (PNG_{t-1} + PNG_{t-2} + PNG_{t-3}/3)]\} \quad (15)$$

where σ_0 and σ_G are estimated variances of the error terms associated with the equations of average discovery size of oil and natural gas respectively.

The determination of the average size of discovery for crude oil and natural gas depends upon economic as well as geologic variables affecting oil prospects. Fisher's [9] study of wildcat drilling and discovery has shown that for economic as well as geologic reasons, small prospects considered by operators tend to be relatively certain and large prospects relatively risky. This is because large prospects, by offering larger potential returns on investment, attract operators willing to take higher levels of risk than do small prospects. Fisher also suggests that there are many more small prospects than large prospects to be discovered. Hence, small price increases should greatly increase the number of small prospects deemed worth drilling. He has further argued that operators prefer to drill the smaller but less risky prospects as prices increase. Under such conditions an increase in price may actually decrease the average size of prospects which are drilled.¹⁰ Even though higher oil prices would be expected to result in more drilling activity in the short-run, the average discovery size is likely to decrease over time because the finite stock of oil and gas is depleted.¹¹

The search for oil and gas is carried out jointly. Because of their common occurrence, new oil discoveries are associated with discoveries of natural gas. The higher the ratio of past gas discoveries to past oil discoveries the higher will be the probability of finding oil. The

discovery of large gas fields may act as an incentive for the drilling of large but potentially risky prospects.

The equations representing the average size of discovery for oil ($ADSZ_t$) and gas ($SZNG_t$) are:

$$ADSZ_t = f(ADSZ_{t-1}, SUC_{t-1}, SZNG_{t-1}, P_t, PNG_t) \quad (16)$$

and

$$SZNG_t = f(SZNG_{t-1}, SUC_{t-1}, ADSZ_{t-1}, P_t, PNG_t) \quad (17)$$

where (SUC) is the success ratio defined as the ratio of productive to total new wells drilled, and (P), (PNG) are the price of crude oil and natural gas at the well head, respectively.¹² In turn, the success ratio at any time period is a function of the success ratio in the previous time period, the average size of discovery for crude petroleum (ADSZ) in the previous period, the average size of discovery of natural gas (SZNG) in the previous period and the depth at which exploration is taking place. Hence the following relationship is estimated in order to complete the model:

$$SUC_t = f(SUC_{t-1}, ADSZ_{t-1}, SZNG_{t-1}, DEP_t) \quad (18)$$

where (DEP) is the average depth of new exploratory wells.

Thus the structural model to be estimated consists of eleven behavioral equations corresponding to equations (3), (4), (5), (6), (7), (9), (10), (15), (16), (17) and (18) in this section. Finally, three identities (1), (8) and (11) complete the model.

Statistical Results and their Interpretation

The estimated model consists of 11 stochastic equations and 3

identities. Several of the endogenous variables are simultaneously determined and both two and three stage least squares (3SLS) estimation methods were used. The three stage least squares coefficients presented had standard errors somewhat lower than the two stage least squares coefficients. The resulting estimates are summarized in Table 1, while data sources, transformations and the variables utilized are summarized in Table 2.

Coefficients for most parameters estimated via 3SLS were substantailly larger than the respective standard errors and signs agreed with hypothesized results throughout the model.

Expected higher prices constitute an incentive to increase exploratory drilling for the discovery of crude oil through the expected higher profits implied. Furthermore the effect of the success ratio in $t-1$ (SUC_{t-1}) in equation (4) is negative, because of the inventory depletion effect, and positive in equation (3) because of the "incentive-toward-larger-prospects effect."¹³ The success ratio furthermore is positively related with depth in that the deeper the exploratory wells are dug the larger the expected success ratio tends to be.

The average size of natural gas discoveries (SZNG) becomes important in crude oil exploratory because the two products are jointly produced. Prior discoveries of natural gas ($SZNG_{t-1}$) indicate possibility of finding crude oil.¹⁴ Since large prospects and certainty tend to be inversely related, $SZNG_{t-1}$ must be positively related to the success ratio in equation (4). But, because of inventory depletion the average size of natural gas discoveries

TABLE 1.--THREE STAGE LEAST SQUARES ESTIMATE OF U.S. OIL PRODUCTION MODEL, 1959-1972

New exploratory wells

$$(1) \text{TED}_t = 77747.5 - 69778.8 \text{ACW}_t - 2581.9 \text{INT}_t + 0.0066 [4a_0^2 \text{ADSZ}_t^2 (P_{t-1} + P_{t-2} + P_{t-3})^2/9 + 4a_1^2 \text{SZNG}_t^2 (\text{PNG}_{t-1} + \text{PNG}_{t-2} + \text{PNG}_{t-3})^2/9] \\ (3142.4) \quad (6577.0) \quad (562.2) \quad (0.0014) \\ - 0.00114 [\text{ADSZ}_t (P_{t-1} + P_{t-2} + P_{t-3}/3) + \text{SZNG}_t (\text{PNG}_{t-1} + \text{PNG}_{t-2} + \text{PNG}_{t-3}/3)] \\ (0.00025)$$

Average discovery size of oil

$$(2) \ln \text{ADSZ}_t = -74.75 - 0.0815 \ln \text{ADSZ}_{t-1} + 15.30 \ln \text{SUC}_{t-1} - 1.45 \ln \text{SZNG}_{t-1} - 0.341 \ln P_t - 1.31 \ln \text{PNG}_t \\ (12.8) \quad (0.0353) \quad (2.41) \quad (0.08) \quad (0.449) \quad (0.27)$$

Average discovery size of gas

$$(3) \ln \text{SZNG}_t = -54.29 - 0.245 \ln \text{SZNG}_{t-1} + 10.08 \ln \text{SUC}_{t-1} - 0.0235 \ln \text{ADSZ}_{t-1} + 0.911 \ln \text{PNG}_t + 1.74 \ln P_t \\ (29.93) \quad (0.176) \quad (5.19) \quad (0.0078) \quad (0.819) \quad (3.84)$$

Success ratio

$$(4) \ln \text{SUC}_t = 2.48 - 0.432 \ln \text{SUC}_{t-1} - 0.0148 \ln \text{ADSZ}_{t-1} - 0.0148 \ln \text{SZNG}_{t-1} + 0.610 \ln \text{DEP}_t \\ (0.60) \quad (0.140) \quad (0.0022) \quad (0.0046) \quad (0.043)$$

Extensions of reserves

$$(5) \ln \text{EC}_t = 5.82 - 0.0623 \ln \text{DC}_{t-1} - 0.201 \ln \text{DC}_{t-2} + 9.83 \ln [1.05 (0.75 P_{t-1} + 0.2 P_{t-2} + 0.05 P_{t-3})] \\ (1.64) \quad (0.0346) \quad (0.035) \quad (1.10)$$

Revisions of reserves

$$(6) \text{RC}_t = 1018942.79 + 0.095 \text{AR}_{t-1} \\ (166053.42) \quad (0.033)$$

Production out of reserves

$$(7) S_t = 26796754.3 + 14919181.5 \ln [1.05 (0.5 P_{t-1} + 0.3 P_{t-2} + 0.2 P_{t-3})] + 45.05 \text{TR} - 1222577.01 [1.05 (0.35 (\text{PRO}_{t-1} - \text{INT}_{t-1}) + 0.2 (\text{PRO}_{t-3} \\ (4744257.7) \quad (3783596.7) \quad (5.80) \quad (80744.7) \\ - \text{INT}_{t-3}) - 0.2 (\text{PRO}_{t-4} - \text{INT}_{t-4})]$$

Imports of crude oil

$$(8) \ln M_t = -18.58 + 1.15 \ln M_{t-1} + 1.26 \ln S_t - 0.299 \ln P_t - 4.28 \ln \text{REF}_t \\ (2.88) \quad (0.12) \quad (0.30) \quad (0.185) \quad (0.65)$$

Addition of natural gas liquids

$$(9) \ln \text{NG}_t = 12.74 - 0.0976 \ln (P_t / \text{PNG}_t) + 0.332 \ln T^2 \\ (0.583) \quad (0.0590) \quad (0.026)$$

Processing Gain

$$(10) \ln \text{GA}_t = -36.49 - 3.93 \ln \text{NG}_t + 6.39 \ln S_t + 1.95 \ln T^2 \\ (32.65) \quad (2.61) \quad (3.66) \quad (0.46)$$

Price of crude oil

$$(11) P_t = -5.25 - 0.0000013 \text{PNG}_t - 0.120 \text{REF}_{t-1} + 0.0000015 [1.1 (0.65 \text{DISTR}_{t-1} + 0.35 \text{DISTR}_{t-2})] + 0.702 \text{PM}_t \\ (1.07) \quad (0.0000008) \quad (0.028) \quad (0.0000008) \quad (0.102)$$

$$(12) \text{DC}_t = \text{ADSZ}_t \times \text{TED}_t$$

$$(13) \text{TR}_t = \text{R}_{t-1} + \text{DC}_t + \text{EC}_t + \text{RC}_t$$

$$(14) \text{DISTR}_t = S_t + M_t + \text{NG}_t + \text{GA}_t^{**}$$

* (standard errors in parentheses)

** This identity serves as a link of this model with the demands and supplies of refined products.

TABLE 2.--LIST OF VARIABLES AND DATA SOURCES

TED	= number of new exploratory wells drilled (total productive and dry holes drilled each year). Source: [3].
SUC	= success ratio (ratio of productive to total new wells drilled).
ADSZ	= average size of new oil discoveries (ratio of new discoveries to total productive and dry holes).
SZNG	= average size of new natural gas discoveries (ratio of new discoveries to total productive and dry holes) [3].
DC	= new oil discoveries, measured in 42-gallon barrels [4].
EC	= extensions of oil reserves, in 42-gallon barrels [4].
TR	= total reserves, beginning of year (in 42-gallon barrels) [4].
DEP	= average depth of new exploratory wells (in feet) [3].
EX	= expenditures for exploration and drilling (in dollars) [3].
ACW	= average cost per exploratory well drilled (in dollars) [3].
R	= crude petroleum reserves (proved reserves at the end of year), measured in 42-gallon barrels [4].
PNG	= price of natural gas liquids at the well head (dollars per barrel) [26].
P	= price of crude oil at the well head (dollars per barrel) [26].
S	= production of crude oil (thousands of 42-gallon barrels) [3].
PRO	= profit rate on equity of petroleum industry [10].
INT	= interest rate (price of commercial paper 4 to 6 months) [8].
M	= imports of crude petroleum (S.I.T.C.: 331.01). Figures converted to thousands of 42-gallon barrels from metric tons [25].
PM	= import unit price. Import value (f.o.b.) reported in [25].
REF	= refining capacity utilization. Refining capacity found in [3].
NG	= natural gas liquids added (thousands of 42-gallon barrels) [3].
GA	= processing gain (thousands of 42-gallon barrels) [3].
T	= linear time trend.
DISTR	= sum of domestically supplied refined products, net of imports exports and change in petroleum stocks (42-gallon barrels) [26].
RC	= revisions of established reserves (42-gallon barrels) [4].

decreases over time as does the average size of crude oil discoveries.

Extensions of crude oil reserves (EC) respond positively to increased expected prices which make secondary recovery methods more profitable, especially if recent additions to reserves (DC) are relatively small. Production out of reserves (S) also responds positively to expected prices, the amount of proven reserves and the relative profitability of the petroleum industry. The inclusion of current price levels rather than expected prices in an alternative formulation of the model [19] did not yield significant results. The implication of both the theoretical framework and the empirical results is that the prevailing price during each time period may or may not be positively related to production out of reserves. Production out of reserves increases mainly in response to a sustained rise in prices.

Imports of crude petroleum are as expected inelastic with respect to the import price (the elasticity is 0.21) partly because of the quota system existing over the sample period. The price of crude responds not only to the expected demand for refined products but also to the capacity utilization in refining.

Model Validation and Prediction

An extensive number of validation measures were calculated to evaluate the efficacy of the model as a predictive device within the sample period. Values for key validation measures are presented in Table 3. The comparatively low Root Mean Square Errors for all

equations suggest that the model could reproduce sample data with a high degree of accuracy. Both the original Theil coefficient [23] and the New Theil coefficient were calculated [24 p. 28] with similar results (See Table 3, Theil Coefficient No. 1 and No. 2 respectively). The Theil coefficients were near zero for all equations further substantiating the efficacy of the model within the sample period. Regression coefficients of actual on predicted values were also near one. The correlations between actual and predicted values were high for all equations except equations (3) and (6) of the model.

To further evaluate the accuracy of the econometric model for policy purposes, predictions for three important variables generated by the model (domestic production out of reserves, the price of crude oil at the wellhead and crude oil proven reserves) were simulated using data not only from within (1959-1972) but also beyond the sample period (1973-1976). Even though the data for the period 1973-1976 was not used in the derivation of coefficients in the model, the simulated results reveal that the model will predict for years beyond the sample period with a high degree of accuracy (Table 4).¹⁵ For most equations the model predicts a high proportion of the turning points over the period 1959-1976.

Summary

We have developed an econometric model which examines the responsiveness of petroleum exploration to economic incentives. The model consisted of 11 stochastic equations and 3 identities.

The model was estimated via three stage least squares and was validated with a variety of numerical measures. Behavior of the model beyond the sample period appeared to be quite good. Statistical results suggest that producers respond to expected sustained price increases, and that if our nation's proven reserves of oil are to increase, we must be willing to pay a higher price.

Table 3.--Validation of Three Stage Least Squares Model

Equation No.	Root Mean Square Error	Thiel Coefficient No. 1 ^a	Thiel Coefficient No. 2 ^b	Regression Coefficient of Actual on Predicted	ρ_{PA}^c
(1) New exploratory wells	3366 wells/year	.0963	.0963	.937	.94
(2) Average discovery size of oil	1253 barrels/ well drilled/ year	.0830	.0831	.988	.97
(3) Average discovery size of gas	1.704 trillion cu ft/yr	.0753	.0755	.962	.34
(4) Success ratio	10.11 percent/yr.	.0021	.0019	.939	.94
(5) Extension of reserves	1155 barrels/yr.	.0105	.0105	.950	.92
(6) Revisions of reserves	1312 barrels/yr.	.0193	.0193	.838	.62
(7) Production out of reserves	85,700,000 barrels/yr.	.0282	.0283	1.010	.97
(8) Imports of crude oil	1068 barrels/yr.	.0051	.0051	.991	.95
(9) Addition of natural gas liquids	1066 barrels/yr.	.0048	.0048	.967	.97
(10) Processing gain	1892 barrels/yr.	.0572	.0572	1.006	.88
(11) Price of crude oil	1.015 dollars/yr.	.0052	.0052	1.000	.98

$$a \sqrt{\frac{1}{n} \sum (P_i - A_i)^2}$$

$$\sqrt{\frac{1}{n} \sum P_i^2} + \sqrt{\frac{1}{n} \sum A_i^2}$$

$$b \sqrt{\frac{\sum (P_i - A_i)^2}{\sum A_i^2}}$$

$$c \frac{\frac{1}{n} \sum (P_i - \bar{P}) (A_i - \bar{A})}{\sqrt{\frac{1}{n} \sum (P_i - \bar{P})^2 \frac{1}{n} \sum (A_i - \bar{A})^2}}$$

where:

P_i = ith predicted value

A_i = ith actual value

Table 4.--Historic Simulation and Projection of Selected Variables

Year	Crude oil reserves (revisions, extensions and new discoveries) ^a		Price of crude oil at the well head ^b		Domestic production out of reserves ^a	
	Actual	Simulated	Actual	Simulated	Actual	Simulated
1959	3,666.7	3,427.7	2.90	2.99	2,574.6	2,417.3
1960	3,365.3	2,340.3	2.87	2.94	2,574.9	2,581.5
1961	2,657.5	2,527.9	2.89	2.96	2,621.7	2,529.6
1962	2,180.9	2,320.0	2.90	2.95	2,676.2	2,641.4
1963	2,174.1	2,343.8	2.89	2.94	2,752.7	2,662.8
1964	2,664.7	2,434.5	2.88	2.91	2,786.8	2,972.6
1965	3,048.0	3,272.9	2.86	2.94	2,848.5	3,074.2
1966	2,964.0	3,162.3	2.88	2.94	3,027.7	3,169.6
1967	2,962.1	2,951.2	2.92	3.00	3,215.7	3,179.5
1968	2,454.6	2,282.2	2.94	3.04	3,329.0	3,178.2
1969	2,120.0	2,258.5	3.09	3.06	3,371.7	3,494.2
1970	12,688.9	10,163.9	3.18	3.17	3,517.4	3,387.7
1971	2,317.7	2,277.3	3.39	3.50	3,453.9	3,417.6
1972	1,557.8	1,459.0	3.39	3.50	3,459.0	3,447.0
1973 ^c	2,145.8	2,100.2	3.89	4.19	3,360.9	3,287.7
1974 ^c	1,993.5	2,000.0	6.74	7.03	3,202.6	3,213.7
1975 ^c	1,318.4	1,312.3	7.56	7.87	3,052.0	3,188.0
1976 ^c	3,094.2	3,121.1	8.13	9.69	2,825.2	2,832.8

^aFigures are in ten thousand barrels.^bFigures are in dollars^cProjections

FOOTNOTES

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- ¹The results presented in this study are a portion of a larger and as yet unfinished effort to explain the behavior of the U.S. petroleum industry that combines the exploration and production of crude oil with the production of refined petroleum products to form a complete system.
- ²A number of econometric studies of the domestic petroleum supply have appeared recently. Fisher [9] was the first to estimate supply equations for the U.S. petroleum industry. The influence of Fisher's model is evident in subsequent empirical studies. This model has been further utilized and amplified with reference to crude oil supply by Erickson, Millsaps and Spann [7] and Epple [6]; and with emphasis on natural gas supply by Khazzoom [13] and MacAvoy and Pindyck [15, 16].
- ³For a more detailed description of the technical aspects of crude oil production, see Epple [6, pp. 5-11].
- ⁴Items left out of the identity, such as exports of crude petroleum, change in stocks, etc., constitute less than one percent of the total amount of refined products.
- ⁵This argument was first made by MacAvoy and Pindyck [15] for natural gas production. The same underlying assumptions held in the case of crude oil production. A reserve-production ratio of about 8:1 is usually maintained by oil producers [20]. Hence, an increase of approximately eight million barrels of recoverable reserves is usually needed to maintain a one million barrel increase in production. But, since present technology recovers only about 40 percent of total proven reserves, a discovery of 20 million barrels of oil would actually be needed to increase production by one million barrels.
- ⁶The planning horizon coincides with the depletion of the resource. The equilibrium path that the price of a nonrenewable resource should follow to the point of exhaustion is such that net price is increasing exponentially at a rate corresponding to the interest rate, unless technological progress affects production [6, 12, 18, 22].
- ⁷See Adelman [1, 2].
- ⁸This formulation is appropriate for a decreasing reserve-production ratio. MacAvoy and Pindyck provide evidence concerning the exponential nature of the function [15, 16].

⁹The form of the distributed lag utilized is the following [11]:

$$EP_t = \beta [w_0 P_{t-1} + w_1 P_{t-2} + w_2 P_{t-3}] + u_t$$

where:

$w_i = \beta_i / \sum_j \beta_j$ and $\sum_i w_i = 1$. Furthermore, u_t is a random variable.

¹⁰For the derivation of the above expressions see MacAvoy and Pindyck [16, pp. 67-71 and 132-133].

¹¹This conclusion has been supported by Erickson, Millsaps and Spann [7].

¹²See Erickson, Millsaps and Spann [7] and MacAvoy and Pindyck [16] on the derivation of the joint supplies of oil and natural gas.

¹³This point comes more clearly across in this study, because of the district-distinguishing effects, involved in Fisher's study [9], as Fisher argues the effect of inventory depletion is to reduce the number of small prospects that would otherwise be drilled so that the average discovery size increases. A rise in SUC_{t-1} is accompanied by a fall in SUC_t .

¹⁴Previous studies have obtained negative coefficients for this same variables because natural gas became a valuable by product only after mid-fifties.

¹⁵The discrepancy between the actual and the predicted price of crude oil at the wellhead is due to the regulations imposed by the Federal Government in 1975 which fixed the price of "old" oil to a "ceiling" price of \$5.25 dollars per barrel. "Old" oil constituted about two thirds of the 1975 production and refers to the output produced in excess of output in the same month of 1972. "New" oil and oil from wells that produce less than ten barrels per day are not subject to price regulations.

The actual average price of crude has been increasing as the industry adjusted its production to take advantage of the new regulations. But, the simulated equilibrium price has been increasing at a faster rate primarily due to its relationship with the average import price.

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