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CHANGES IN REAL INCOME AND THE DEMAND FOR DOMESTIC AND FOREIGN SECURITIES IN A SYNTHESIZED EXCHANGE RATE MODEL: AN ALTERNATIVE VIEW

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CHANGES IN REAL INCOME AND THE DEMAND FOR DOMESTIC AND FOREIGN SECURITIES IN A SYNTHESIZED EXCHANGE RATE MODEL: AN ALTERNATIVE VIEW

Abstract

By

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It is typically assumed that real income enters the demand function for securities in a negative fashion [see Turnovsky and Kingston (1977), Levin (1980), Penati (1983), and Kawai (1985)]. This paper shows that this assumption violates the sufficiency condition for stability and is not consistent with earlier portfolio theories. These issues are examined within the context of a synthesized Keynesian and portfolio balance approach.

CHANGES IN REAL INCOME AND THE DEMAND FOR DOMESTIC AND FOREIGN SECURITIES IN A SYNTHESIZED EXCHANGE RATE MODEL: AN ALTERNATIVE VIEW

Introduction

In the application of portfolio balance theory to open economies, it is generally assumed that real income enters the demand function for securities in a negative fashion [see Turnovsky and Kingston (1977), Levin (1980), Penati (1983), and Kawai (1985)]. The purpose of this paper is to show that this assumption violates the sufficiency condition for stability and also contradicts the earlier portfolio theories of Patinkin (1965) and McKinnon (1969). Section one specifies a model which incorporates elements of Keynesian and portfolio balance approaches similar to the models developed by Levin (1980), Frenkel, Gylfason, and Helliwell (1980), Gylfason and Helliwell (1983), Ahtiala (1984), Branson (1986), and Dornbusch (1986). Stability conditions are examined in section two, and the implications of the analysis are presented in section three.

1. A Synthesized Keynesian and Portfolio-Balance Model

The dynamics of a single-country economy can be described by a system of ordinary differential equations:

$$\dot{X} = (dx_i(t)/dt) = H(X(t),a), \quad i=1,3 \tag{1}$$

where $\dot{X}$ is a time (t) derivative of X, and "a" is a k-dimensional vector of parameters. X is a three-dimensional Cartesian space of real income (x_1=Y), interest rate (x_2=r), and exchange rate (x_3=E). This system is a small open economy model which originates from the IS-LM analytical framework of Fleming (1962) and Mundell (1968). The above linear vector fields of X are simultaneously specified in the commodities, domestic and foreign bond markets.

Based on the producers' responses to unexpected inventory changes, a goods market excess demand function $[h_1(X,a)]$ is:

$$\dot{Y} = \alpha[\, C(Y) + I(r) + G + x(Y,E) - Y], \tag{2}$$

where C(Y) is the consumption function which depends positively on real income $[C_Y>0]$. The investment function I(r), is a negative function of the real interest rate (r) $[I_r<0]$. Net exports, x(Y,E), decline with real income (Y), and rise with the exchange rate (E)--the domestic currency price of foreign exchange $[x_Y<0,\ x_E>0]$. Finally, α is an adjustment parameter such that $1 \geq \alpha > 0$.

In the financial sector, private citizens hold a portfolio of domestic real money balances (M), home bonds (B^d), and foreign denominated securities (F),

$$w = M + B^d + EF, \tag{3}$$

Here, w is real wealth, and B^d is the portion of domestic securities (B) held by home residents. Equilibrium in this sector requires that excess demand for all financial assets be equal to zero. This condition must therefore be met both in the (flow) goods market and in the (stock) financial markets (Foley, 1975). Hence, using the wealth constraint (equation 3), the market clearing condition for real balances can be dropped.

The excess supply in the home security market, $[h_2(X,a)]$, can be modelled as:

$$\dot{r} = \beta[B - b^d(r,r^*,Y)w - b^f(r,r^*,Y^*)Ew^*] \tag{4}$$

$$1 > \beta > 0, \quad b^d_r \text{ and } b^f_r > 0, \quad 1 > b^d > 0, \quad 1 > b^f > 0$$

where β is an adjustment parameter. The terms $b^d(r,r^*,Y)$ and $b^f(r,r^*,Y^*)$ denote the domestic and foreign-desired fraction of domestic (w) and foreign (w^*) wealth, respectively, held in domestic securities. Given a single-country economy assumption, the foreign interest rate (r^*), real income (Y^*), and wealth (w^*) are determined exogenously.

Finally, the excess demand function in the foreign security market, $[h_3(X,a)]$, is given by

$$\dot{E} = \lambda[b^*(r,r^*,Y)w - EF] \tag{5}$$

$$1 > \lambda > 0, \quad b^*_r > 0$$

where λ is an adjustment parameter. $b^*(r,r^*,Y)$ denotes the domestic desired fraction of domestic wealth held in foreign securities. EF is the domestic currency supply of foreign securities. The discussion of how real income enters the demand function for securities will be addressed in the following sections.

The wealth constraint implies that the summation of the desired fractions of domestic wealth--those held in real cash balances, domestic, and foreign securities--total one. Hence

$$m(r,Y) + b^d(r,r^*,Y) + b^*(r,r^*,Y) = 1 \tag{6}$$

where m(r,Y) describes the desired fraction of wealth held in domestic real balances.

Therefore, the dynamics of a single country economy can be reduced to the following set of equations:

$$\dot{Y} = \alpha[C(Y) + I(r) + G + x(Y,E) - Y] \qquad (2)$$

$$\dot{r} = \beta[B - b^{d}(r,r^{*},Y)w - b^{f}(r,r^{*},Y^{*})Ew^{*}] \qquad (4)$$

$$\dot{E} = \lambda[b^{*}(r,r^{*},Y)w - EF] \qquad (5)$$

2. Stability Conditions

A solution to the differential equation system (1) is X^{*}, such that $H(X^{*})=0$. Accordingly, the equilibrium level of real income, interest rate, and exchange rate are determined when equations (2), (4), and (5) are simultaneously set to zero. The necessary and sufficient stability conditions require that the trace of the Jacobian matrix J be negative and that its determinant be positive. If $h_{\ell x_j}$ describes the partial derivative of the ℓ-vector field of H with respect to x_j, then the Jacobian matrix J is

$$(j_{ij}) = (h_{\ell x_j}), \qquad i=\ell,\ j= 1,3.$$

The Jacobian matrix, including the signs of the partial derivatives, is shown in Table 1.

[Table 1 approximately here]

All the principal diagonal elements are negative. Hence their summation, the trace of the Jacobian matrix, is also negative, and the necessary condition for stability is met. Assuming a negative income effect in the demand for securities, however, also results in a negative determinant of the Jacobian matrix which violates the sufficiency condition for stability. This can be seen by expanding the Jacobian matrix along its first column. Thus,

$$\det(J) = j_{11}[j_{22}j_{33}-j_{32}j_{23}] - j_{21}[j_{12}j_{33}-j_{32}j_{13}] + j_{31}[j_{12}j_{23}-j_{22}j_{13}].$$

If the partial derivatives in the above expansion are replaced with their signs, then,

$$\det(J) = -[+\ -] - [+\ +] - [+\ +]$$

which after multiplying the signs within the brackets, yields:

$$\det(J) = \quad -\ + \quad -\ - \quad -\ -$$

Following Branson, Halttunen, and Masson (1979, p. 305) by applying the assumption of gross substitution between home and foreign securities, the first term on the right hand side of the above equation can be shown to be larger in absolute value than the second term. Thus det(J) is negative.

On the other hand, if real income enters the demand function for domestic and foreign securities with a positive sign, the determinant becomes positive since det(J) can now be written as:

$$\det(J) = -[+\ -] + [+\ +] + [+\ +]$$

which after multiplying the signs within the brackets yields:

$$\det(J) = \quad -\ + \quad +\ + \quad +\ + \qquad (7)$$

Here five of the six terms are positive. In addition the first (negative) term is likely to be smaller in absolute value than the sixth (positive) term. This follows by assuming, as shown in the appendix, that the goods market clearing condition (GG curve), should have a steeper slope than the foreign security market clearing condition (FF curve), Figure 1a. This occurs if the bond market is more responsive than the goods market to changes in the financial variables such as exchange rate. Equilibrium in all markets would be restored when a given rise in real income requires the exchange rate to rise less in the foreign security market than in the goods market. Thus, the det(J) is positive, and the sufficient condition is met. If this slope condition is not met, the system is unstable (Figure 1b).

[Figures 1a and 1b approximately here]

3. Implications

The previous section demonstrates that the sufficient stability condition is violated when real income is assumed to enter the demand functions for securities in a negative fashion. This section argues that this assumption (restriction) also contradicts the earlier portfolio theories of Patinkin (1965) and McKinnon (1969) for example.

The acceptance of the negative income effect restriction can be seen by taking the partial derivative of equation (6) with respect to real income.

$$m_Y + b_Y + b_Y^* = 0 \tag{8}$$

It is then argued that if real income enters the demand for real balances in a positive fashion, it must enter the demand for securities in a negative fashion (see equation (8)). In support of the above contention, Kawai (1985) cites Patinkin (1965), Tobin (1969), and Foley and Sidrauski (1971).[1] Both Tobin's (1969) and Foley and Sidrauski's (1971) specification for bond demand reflect this restriction.[2] However, in his mathematical appendix, Patinkin (1965) specifies an excess demand function for bonds with an ambiguous income effect.[3] When it comes to the demand function itself, though, he clearly assumes that an increase in real income will "... shift the whole demand function to the right [p. 215]." McKinnon's (1969) Keynesian portfolio balance model, while different from Patinkin's (1965) goods market model, also specifies positive income effect in the demand for securities.

Therefore the assumption that real income enters the demand function for securities in a negative fashion is based on a result contingent upon a particular specification of the portfolio balance model.[4] One remedy for the implicit instability problem arising from the negative income effect, is to drop real wealth from the demand function, as suggested by McKinnon (1969).[5] Alternatively, have real wealth only enter the demand function indirectly through its elements, such as real cash balances (Patinkin, 1965).

To sum up, real income should enter the demand function for bonds in a positive fashion as has been previously assumed in the closed economy portfolio balance models. If a negative income effect is assumed, the sufficiency condition for stability is not met. This would make comparative static analyses less reliable since the set of unstable equilibria may no longer be empty.

FOOTNOTES

1- Footnote 4, p. 394.

2- Foley and Sidrauski (1971), section 3-1.

3- Patinkin (1965), p. 484.

4- Even if one included real wealth in the demand functions, $b(r,r^*,Y,w)$, (Jones, 1969) given the wealth constraint, equation (8) still holds and the negative income effect must results.

5- McKinnon argues that "Under the static assumptions, Y and i [interest rate] together give a measure of permanent income and a discount rate; and hence provide a measure of real wealth (including human wealth) into the foreseeable future [p. 212]."

REFERENCES

Ahtiala, P., "A Synthesis of the Macroeconomic Approaches to Exchange Rate Determination," European Economic Review, 24, No. 2 (March 1984): 117-136.

Branson, W., "The Limits of Monetary Coordination As Exchange Rate Policy," Brookings Papers on Economic Activity, 1 (1986): 175-94.

Branson, W.H. Halttunen, H. and Masson, P., "Exchange Rates in the Short Run: the dollar-Deutschemark Rate," European Economic Review, 10, No. 3 (December 1977): 303-24.

Dornbusch, R., "Flexible Exchange Rates and Excess Capital Mobility," Brookings Papers on Economic Activity, 1 (1986): 209-26.

Fleming, M., "Domestic Financial Policies under Fixed and under Flexible Exchange Rates," Staff Papers, NO. 9 (1962): 369-80.

Foley, D., "On Two Specifications of Asset Equilibrium in Macroeconomic Models," Journal of Political Economy, 83, No. 2 (April 1975): 303-24.

Foley, D.K. and Sidrauski, M., Monetary and Fiscal Policy in a Growing Economy. New York: Macmillan, 1971.

Frenkel, J. Gylfason, T. and Helliwell, J., "A Synthesis of Monetary and Keynesian Approaches to Short-Run Balance-of-Payments Theory," Economic Journal, 90, No. 359 (September 1980): 582-92.

Gylfason, T. and Helliwell, J., "A Synthesis of Keynesian, Monetary, and Portfolio Approaches to Flexible Exchange Rates," Economic Journal, 93, No. 312 (December 1983): 820-31.

Jones, R.W., "Comments: Portfolio Balance and International Payments Adjustment," in R.A. Mundell and A.S. Swoboda (eds)

"Monetary Problems of the International Economy," Chicago: University of Chicago Press, 1969.

Kawai, M., "Exchange Rates, the Current Account and Monetary-Fiscal Policies in the Short Run and in Long Run," Oxford Economic Papers, 37, No. 3 (September 1985): 391-425.

Levin, J., "The Dynamics of Stabilization Policy under Flexible Exchange Rates: A Synthesis of Asset and Neoclassical Keynesian Approach," Oxford Economic Papers, 32, No. 3 (November 1980): 411-427.

McKinnon, R.I., "Portfolio Balance and International Payments Adjustment," in R.A. Mundell and A.K. Swoboda (eds) "Monetary Problems of the International Economy," Chicago: University of Chicago Press, 1969.

Mundell, R., "Capital Mobility and Stabilization Policy under Fixed and Flexible Exchange Rates," in R.E. Caves and H.G. Johnson (eds) "Readings in International Economics," Homewood: Illinois: Richard D. Irwin, Inc., 1968.

Patinkin, D., Money, Interest, and Prices. New York: Harper and Row, 1965.

Penati, A., "Expansionary Fiscal Policy and the Exchange Rate: A Review," Staff Papers, 30, No. 3 (September 1983): 542-569.

Tobin, J., "A General Equilibrium Approach to Monetary Theory," Journal of Money, Credit, and Banking, 1, No. 1 (February 1969): 15-29.

Turnovsky, S.J. and Kingston, G.H., "Monetary and Fiscal Policies under Flexible Exchange Rates and Perfect Myopic Foresight in an Inflationary World," Scandinavian Journal of Economics, 79, No. 4 (1977): 424-441.

APPENDIX

The purpose of this appendix to show that, if the sixth term in equation (7) is greater in absolute value than the first term, then the GG curve slopes steeper than the FF curve. det(J) would then be positive, and the system would be stable (Figure 1a).

The first term of equation (7), $j_{11}j_{22}j_{33}$, is smaller in absolute value than the third term, $-j_{31}j_{22}j_{13}$, if:

$$j_{11}j_{22}j_{33} - j_{31}j_{22}j_{13} > 0 \qquad \text{(A-1)}$$

By factoring (the negative) j_{22} from the above equation, the (A-1) condition can be reduced to the following:

$$j_{11}j_{33} - j_{31}j_{13} < 0 \qquad \text{(A-2)}$$

The (A-2) condition can be rearranged to derive the following condition:

$$-(j_{31}/j_{33}) < -(j_{11}/j_{13}) \qquad \text{(A-3)}$$

The left and right-hand sides of the (A-3) condition are the partial derivatives of exchange rate with respect to real income in the foreign security and (home) goods market clearing conditions. They can be rewritten as:

$$(\partial E/\partial Y)_{FF} < (\partial E/\partial Y)_{GG} \qquad \text{(A-4)}$$

TABLE 1

Jacobian Matrix J

Partial Derivatives of the ...	With Respect To: Real Income	Interest Rate	Exchange Rate
Goods Market	$[-1 + C_Y + x_Y]$ (−)	$[I_r]$ (−)	$[x_E]$ (+)
Domestic Bond Market	$[-wb^d_Y]$ (?)	$[-wb^d_r - Ew^* b^f_r]$ (−)	$[-Fb^d - w^* b^f]$ (−)
Foreign Bond Market	$[wb^*_Y]$ (?)	$[wb^*_r]$ (−)	$[-(1-b^*)F]$ (−)

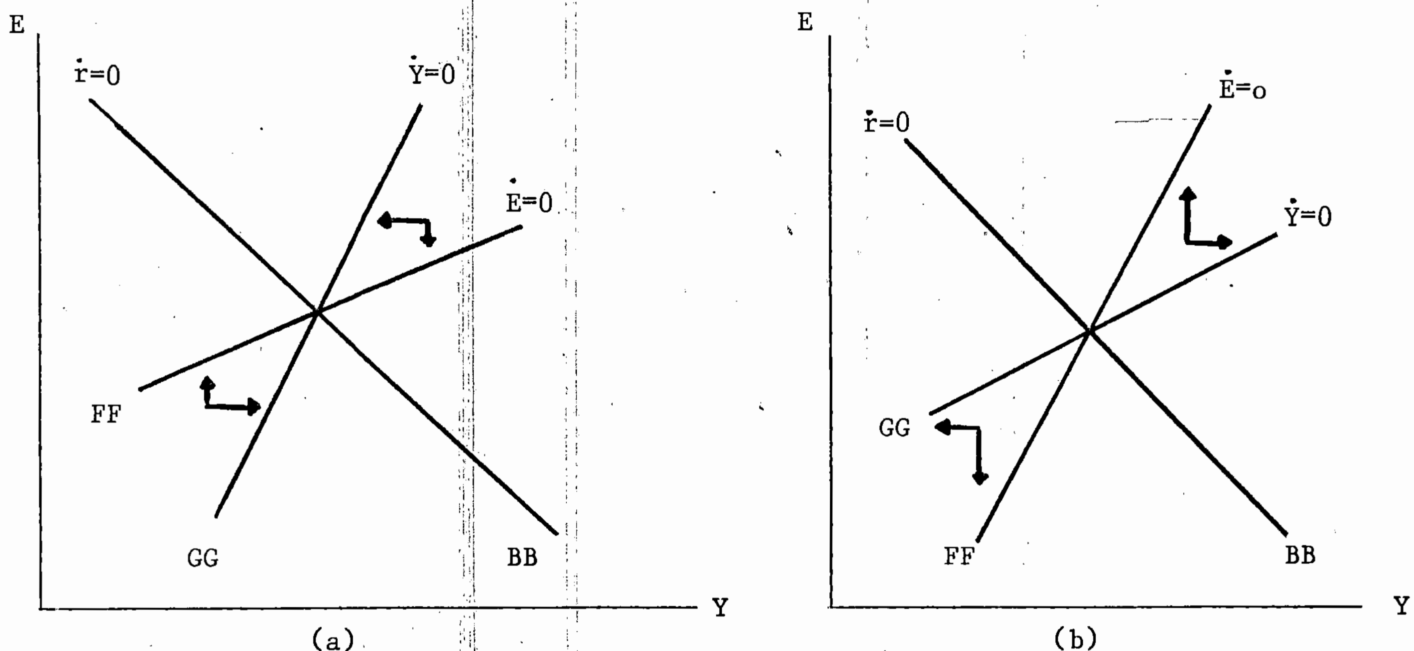

FIGURE 1: GG Curve Illustrates the Goods Market Equilibrium Solution.
BB Curve Illustrates the Home Security Equilibrium solution.
FF Curve Illustrates the Foreign Security Equilibrium Solution.