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EVALUTING LOGICAL PLURALISM

by

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ABSTRACT

Recently some philosophers, in particular J. C. Beall and Greg Restall, have defended a view they refer to as 'logical pluralism'. This is the position that there are, in fact, several equally good but distinct logical systems according to which different arguments come out valid and invalid. No one system, they claim, is any more 'correct' than any other. I will have several criticisms of this view. I first argue that the phenomena of *logical recapture* causes problems for the pluralist. Somewhat roughly a logic is recaptured if, though all its argument forms were not valid in the full language, a restriction on the formulas of the language can render all those argument forms valid.   I argue that once we recognize that this is possible the pluralist will require further argument if she is to contend that her account of the validity of the logic in question is superior to the logician who embraces recapture. My second criticism casts doubt on Beal and Restall's view that any time one specifies a set of truth conditions one has established a type of case in which claims may be true as well as a corresponding type of necessity. I will also make some methodological points about how to decide whether logical pluralism is true and which logic or logics are correct. And finally I will concede that a form of logical pluralism slightly different from the one endorsed by Beall and Restall may indeed be true. But ultimately I will have to leave the question of whether this alternative is indeed a legitimate form of logical pluralism to be settled on another occasion.

I. Introduction

First , I lay out the basic view itself. Then I'll clarify some issues regarding the meaning of 'logic' and the criteria used to evaluate the correctness of a logic. I distinguish several senses of 'logic' and give an account of the 'norms' of logic. Once I've done this I specify several possible kinds of logical pluralism and suggest one of these as, most likely, the kind endorsed by Beall and Restall.

I'll then raise several objections to the pluralist stance. First, I'll address the problem of *logical recapture*. To a first approximation, one logic L1 recaptures another, L2 when the language of L1 can be restricted in such a way that all and only the arguments valid in L2 are valid in L1. Once we recognize that this is possible we have an alternate explanation of the apparent validity of the pluralists logics. Thus, even if we were to accept that Beall & Restall have provided us with prima facie evidence of different sets of valid arguments, further steps would be necessary to establish that logical pluralism is indeed correct. There are two dialectical paths the pluralist might take to defend themselves. They might try to show that no logic does in fact recapture the logics they propose. Another strategy would be to argue that the pluralist explanation of the validity of the logics in question is more plausible than the account on which there is a single, which recaptures the pluralists logics.

Thirdly, I will explore the relationship between the logical and extra-logical realms. First it's important to recognize the role epistemic justification plays here. For

instance, there may be no one set of arguments, which are valid in all logics. If there

were such a set – for instance all arguments taking the form of Modus Ponens – we might

think it uncontroversial to appeal to such arguments as we reason about whether other

arguments are acceptable. If, on the other hand, there is no one form, which all logics

agree on then we may feel we have no firm ground to stand on. But it is at least in

principle possible for one to be epistemically justified in believing that certain arguments

are valid quite apart from whether all currently developed logics declare them valid. In

fact, it is in principle possible that we are a priori justified in accepting that all arguments

of the form Modus Ponens are valid in the same basic a priori way that we are justified in

accepting that $2 + 2 = 4$.

On the other hand, merely saying that an argument seems valid in the same way

as $2 + 2 = 4$ seems correct may not carry a lot of weight with someone who accepts a

logic in which that argument is invalid. The very basicality of the subject matter under

discussion leaves us with a thin evidence base to work from. This is why it may be

advantageous to bring in considerations from other extra-logical subjects as well. For

instance, it is in principle possible that we are more justified in believing some truths in

metaphysics or the theory of meaning than some highly technical logical theses. If it can

be shown that particular theses in either metaphysics or some other extra-logical subject

matter can have an effect on one's logical commitments this will allow us to bring our

intuitions regarding these extra-logical matters to bear on the debate about which logic is

correct. During the course of the paper I will discuss some ways in which this could

happen.   Thus one point of the current paper is a general methodological point to the effect that logic should not, or not exclusively, be studied in isolation.

Next I argue that Beall and Restall's account of cases, and the circumstances under which a distinct type of case can legitimately be said to exist is unclear. In particular some things they say make it seem as if they want to reduce modality itself to truth in cases and yet on other occasions they appear to work with a primitive form of modality which itself constrains cases. I argue that it is implausible to see modality as dependent on truth in cases and thus their views on modality taken as a whole are implausible.

Next, I concede that Beall and Restall may be right in maintaining that there are several distinct kinds of validity and thus that several different systems *may* count as logics. But the different kinds of validity I consider are not the same as those of Beall & Restall's pluralism, arising as they do from different considerations. Ultimately however my conclusion in this regard will have to remain tentative. The question of whether the various possible senses of 'validity' allow the specification of a logic, and thus whether there is more than one legitimate logic, depends partly on the nature and purpose of logic as well as an account of logical form which is yet to be worked out. I do hope, however, to clarify some of the difficulties Beall & Restall's form of pluralism faces as well some issues regarding what exactly the pluralist/monist debate turns on.

In the final section of the paper I address two objections that may be made against the arguments I put forward in the paper. One objection is to the effect that recapture is a merely language relative variation in the arguments that can be declared valid. But Beall and Restall reject language relative differences in consequence as orthogonal to their form of pluralism. I argue that given the phenomenon or recapture and of language relative differences in the consequence relation this objection is either highly implausible or amounts to a new dialectical position on the pluralist's part. In addition it is not clear that pluralism in its current form actually offers an account of differences in the consequence relation that are non-linguistic.

The second objection is to the effect that when I discuss the possibility of quantification into impossible worlds in the last section of the paper I open the door for Beall and Restall's cases to re-enter the debate as impossible worlds if nothing else. I respond by noting that, on the one hand, there are many ways to formulate possible worlds and whether Beall and Restall's cases will be among the kinds recognized by a future logic is an open question. On the other hand, I note that during the course of the paper I argue that an account of logical form is probably necessary for a true account of logical consequence and in this my view would still differ from Beall and Restall's.

II. Logical Pluralism: The Argument

To state Beall and Restall's argument for logical pluralism we must first make clear some

preliminary connections that will be important for making sense of their claims. First

they hold that the most central feature of logic is the consequence relation.

> Logic is about consequence. Logical consequence is the heart
>
> of logic; it is also at the centre of philosophy and many theoretical
>
> and practical pursuits besides. (Beall & Restall 2006 p. 3)

Given that the central notion of logic is the notion of consequence they believe that if

there is more than one consequence relation then there is more than one logic. In fact,

they appear to think that being a pluralist about consequence just is being a pluralists

about logic. As they put it:

> we defend what we call *logical pluralism*, the view that there is
>
> more than one genuine deductive consequence relation . . ." (Beall
>
> & Restall 2006, p. 3; emphasis in original)

They go on to say that, in their view, validity is "the most powerful and

productive idea, constraining the concept of logical consequence . . . providing a dividing

line between the logical and the non-logical . . ." (Beall & Restall 2006 p. 23) They then

offer the following quote from Richard Jeffrey's *Formal Logic*:

Formal logic . . . aims to provide systematic means for telling

whether or not given conclusions follow from given premises, i.e.

whether arguments are valid or invalid . . . (ibid.; quoting Jeffrey

'91 p I)

Continuing:

This analysis of validity will form the centre of our book, and

it is at the heart of pluralism about logical consequence. (ibid.)

Provided that we take a conclusion's following from certain premises to be the

same as it's being a logical consequence the quote from Jeffrey draws a direct connection

between logical consequence and validity. Arguments that are valid are those that have

conclusions that are consequences of their premises.

But so far this is just the set up, the pluralist part of logical pluralism is a result of

the fact that Beall & Restall think that there is more than one distinct kind of validity. To

argue for this they characterize a valid argument thus: "A valid argument is one whose

conclusion is true in every case in which all its premises are true."(Beall & Restall (2006)

p. 23) They then go on to argue that there are several different kinds of cases. Thus there

are several distinct kinds of validity and hence many logics.

According to Beall and Restall, the legitimate types of substituends for cases in the definition of validity are possible worlds, Tarskian models, situations, and constructions. We need not go into all these in detail. In order to establish logical pluralism a pluralist only needs to establish that there are at least two irreducibly different, legitimate kinds of case, which yield different kinds of validity. So I will focus on the more familiar pair of the four cases they invoke, possible worlds and Tarskian models. The intuitive idea of a possible world will, I trust, be familiar enough, despite the variety of more specific accounts of what possible worlds are supposed to be. Models might be familiar to many as well, but I will take a moment to explain the basic idea and in the process lay out what the difference is between the two resulting accounts of validity.

When dealing with models we first need an underlying formal language L with well-formed-formulas (wffs) defined in it; generally by defining atomic formulas and then giving recursive formulas for the logical connectives. Then, naturally, we have a model M, which consists of a non-empty domain D of objects and a function I. The function I assigns to each name an object in the model and to each n-place predicate a set of ordered n-tuples of objects answering to that predicate. An assignment α is a particular way of assigning values to the variables and an x-variant of α is an assignment that assigns all the same values to the variables with the possible exception of x.[1] With

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[1] I say 'possible exception' because technically α is a variant of itself.

these established the following truth conditions could be used to characterize a logic along Tarskian lines.[2]

- If α is an assignment of D-elements to variables then $I_\alpha(x)=\alpha(x)$. If a is a name, $I_\alpha(a)=I(a)$.

- $Ft_1 \ldots Ft_n$ is true in M, α iff $\{I_\alpha(t_1), \ldots I_\alpha(t_n)\} \in I(F)$

- $A \wedge B$ is true in M, α iff A is true in M, α and B is true in M, α

- $A \vee B$ is true in M, α iff A is true in M, α or B is true in M, α

- $\neg A$ is true in M, α iff A is not true in M, α

- $\forall xA$ is true in M, α iff A is true in M, α' for each x-variant α' of α.

- $\exists xA$ is true in M, α iff A is true in M, α' for some x-variant α' of α.

Now, suppose we were to translate from natural language into our selected formal language the sentences "A is red. Therefore, A is colored". One way to do this is to substitute for 'is red' the predicate letter F, for 'is colored' the predicate letter G, and to assign the object referred to by 'A' to 'a'. We then get the following formal sentence, '$F(a) \models G(a)$'.

To evaluate the validity of this argument all we need to do is remember how names and predicates are assigned objects or sets in the model and look at the first two truth conditions outlined. Basically, F(a) is true iff the object assigned to 'a' is an element of the set assigned to 'F'.

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[2] The following truth-conditions were taken directly from Beal & Restall (2006), as an example to demonstrate the difference between a possible worlds and a model theoretic understanding of consequence.

Given our definition of 'validity' if we construe 'cases' as Tarskian models 'F(a) |= G(a)' is not valid. This is because there is nothing in the above specifications of how a language is assigned a model to prevent assignments to 'F' of a set that does not overlap with the set assigned to G in terms of the objects involved. In which case there will be models in which F(a) is true while G(a) is false. And since in testing for validity we must check for truth in all cases we must check the models in which this occurs as well. This, even though, F was 'is red' and G was 'is colored'.

On the possible-worlds understanding of 'case', however, we would get a definition of 'validity' that looks like this: An argument is valid iff in every possible world where the premises are true the conclusion is also true. Understood this way the argument "A is red. Therefore A is colored" comes out valid since, by definition, possible worlds don't include things that are impossible and obviously it's impossible that something is red and not colored. Thus there are at least two different kinds of validity, and Beall and Restall do not think there is any reason for us to declare one of these to be genuinely *logical* validity and deny that the other is. Hence, if there are two equally legitimate kinds of validity and validity determines consequence, then there are two equally good consequence relations. And we saw above that Beall and Restall defined logical pluralism as the view that there is more than one logical consequence relation. So if there are at least two legitimate kinds of cases, then there are at least two logics.

III. 'Logic'

In order to assess the feasibility of this position and in order to be able to even begin to approach the question of which logic or logics are correct it will be instructive to look at the different meanings of 'logic'.

In one sense 'logic' refers to an activity or discipline. In this sense 'logic' refers to the discipline which logicians practice, namely the study of or formulation of logical systems. Stephen Read and Andrew Aberdein go on to divide this sense of "logical endeavor" by making a distinction between different motivations a logician might have.

On the one hand, research in logic can be pursued to improve understanding of reasoning in natural language (or some techni-cal or scientific enrichment thereof): natural argumentation. On the other hand, logic can be a purely formal enterprise, manipulating symbols in accor-dance with explicit rules. We might characterize this as a distinction between 'rough' and 'smooth' logic. (Read & Aberdein (2009) p. 2)[3]

So rough logic might include the study of various formal systems and informal argumentation with the purpose of constructing the formal systems to capture the structure of intuitively correct informal arguments.

Smooth logic on the other hand is a more abstract exploration, which takes various

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[3] The quote is from Read and Aberdein (2009) but they attribute the idea to Goldstein (1992).

formal systems and varies certain features, imposing constraints here, making extensions there, and cataloguing the features that the system gains or loses as a result. As Read and Aberdein put it "purely formal results and applications to mathematics or computer science are obviously smooth . . ."(Read & Aberdein, 2009 p. 2)[4]

However, 'logic' in this sense is an exploration of 'logics' in other senses.   For instance, there are various formal systems like classical or intuitionistic 'logic', which have been developed over the years.   These formal systems can be studied for their own sake. We can investigate to what extent they model various forms of argument which we find intuitively convincing. From here on out I'll refer to these as formal logics.

But, Thomas Hofweber offers a slightly different account of logic as a discipline. He puts emphasis on the fact that 'good reasoning', which we may take to offer intuitively good informal arguments, *follows* 'valid inferences'. He says that in one sense logic is a

> discipline . . . [which] deals with certain valid inferences and
>
> good reasoning based on them. . . . [I]t deals with inferences whose
>
> validity can be traced back to the formal features of the
>
> representations that are involved in that inference . . . (Hofweber,
>
> 2004)[5]

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[4] This is not to suggest that Beall and Restall's might be endorsing pluralism about logic as an activity. They explicitly say they're not (or something pretty close to that), viz., "We grant that we have . . . not [defended] pluralism about 'logic' understood as the study of consequence relations."(Beall & Restall 2006, p. 88)

[5] Beall & Restall leave things somewhat ambiguous between these two senses. On the one hand they say, "Logic is to do with the evaluation of arguments." (Beall & Restall 2006, p. 8) They are clearly concerned with logic as an activity

Notice that this definition puts emphasis on the belief that there is a formal structure in our reasoning and argumentation in virtue of which it is valid; a formal structure that is there independently of whether any formal logics are ever developed which can be taken to represent that structure. So we can make a distinction then between the historical formal systems designed to capture the formal structure of arguments and that formal structure itself present in the arguments in virtue of which they are valid.

So in this sense logic is a discipline which attempts to study and capture valid inferences; the real valid inferences. We might say then that it is the study of logic itself. It is natural when talking about this sense of 'logic' to talk as if there is only one such logic. For instance, it is natural to say that these various formal systems are attempting to characterize logic, or 'the true logic', or 'logic itself'. But even if this is the more natural way of speaking among those who use the terms, it may simply be mistaken.

We can think of this on analogy with ethical theories. Even though it is common among ordinary folks for people to believe that there is just one true moral system, this may legitimately be questioned in a philosophical context. In fact, it's quite common for logic to be taken to be among the normative disciplines along with ethics, so the analogy may not be far off at all. I'll refer to this third sense of 'logic' as 'objective logic'. By

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or discipline here, but it's not clear whether they are thinking of the activity of formulating a formal system of logic or of learning about the "formal features of representations" in actual argumentation that makes it 'valid'. It's possible to achieve the latter by doing the former. But it also seems possible to achieve the latter without doing the former. And since one might design some new formal system without caring whether it imitates the 'formal features' of real valid arguments, it seems one might do the former without the latter as well.

this I mean whatever it is about genuinely valid arguments in virtue of which they are valid and in virtue of which future valid arguments can be made. Perhaps this is a system of, as Hoffweber put it, "formal features of representations".

Given this characterization note that someone could be a pluralist with regard to objective logic as well. Objective logic is more than just the set of genuinely correct arguments. It is the set of arguments that are correct according to some objective feature all the arguments share and which is not shared by any invalid arguments. So one will be a pluralist about objective logic if, roughly, there is no single feature shared by all and only genuinely correct arguments.

IV. Criteria

Now the pluralist claims that there is more than one kind of validity and that each of them is equally correct. But, 'correct' in what sense? It could be correct to choose something because morality proclaims it correct or it could be correct to believe something because one currently has more epistemic justification for believing it than anything else. Each of these is a different criterion of correctness. The use of any of these possible criteria for judging correctness constitutes a different way of being correct. So we need to know which criterion of correctness is intended here. Are we saying that more than one logic is morally correct, or that there is more than one we are epistemically justified in believing, or something else entirely?

Let us suppose it is the moral criterion that is at issue, meaning we should choose that logic which meets the demands of morality. Then Beall and Restall's claim would be that more than one logic is morally correct. If all we mean by this is that one might have a moral obligation to choose logic A at one time and logic B at another this hardly seems controversial. It might be that I have a moral obligation to pass my logic class and this requires that I do my work using logic A because this is the system of logic being taught in the class. Perhaps on another occasion I take a class on alternative logics. Then I may have an obligation to use logic B. It's not plausible that this rather mundane fact is the core claim of logical pluralism.

Could the pluralist mean that we have an epistemic obligation to choose more than one logic? This would hardly amount to a new or interesting position in the philosophy of logic either. I might have most justification for accepting logic A at one time but subsequently gain new evidence which points to logic B. But once we put things in these terms a particular question arises. In what sense might one be justified in 'accepting' a logic? Presumably, I will want to accept a logic because my evidence says it is correct or the closest among it's rivals to being correct? So the epistemic sense doesn't get us anywhere. It merely puts off the question of correctness. The kind of correctness we might be justified in thinking our logic possessed might be the moral sense above. But we've already said that this isn't plausibly what the pluralist has in mind.

Is there some other criterion of correctness that might be at issue? If we think back to the distinction between formal and objective logic we can see another possible

criterion. Recall that in the sense in which I am using the terms a formal logic is a historically developed system of logic like intuitionistic logic or one of the systems of relevance logic whereas objective logic is the logic at work in genuinely valid arguments, whatever this turns out to be, in virtue of which those arguments are 'good'. So perhaps the criterion at issue is the degree to which a formal logic reflects the objective logic. In order to know if this is a plausible interpretation I will need to make clearer what I think objective logic is and what it means for a formal logic to reflect objective logic. I will do that in section V. In section VI I will discuss forms of pluralism using closeness to objective logic as the criterion of correctness, as well as forms using more complex understandings of moral criteria of correctness.

But in order to make clear what an objective logic might be we should first make a distinction between the criterion by which a logic is said to be 'correct' and the correctness of drawing a certain conclusion from certain premises. The former notion is our ultimate object of inquiry, but in order to reach it we will need to discuss the latter kind of correctness. Some philosophers have thought that logics are normative. That is, they think that logic gives us norms for correct thinking. I'll be quoting an example of such a philosopher shortly. But there are plausibly correct ways of thinking that involve reasoning to conclusions that are only probable to some degree given our premises and there may also be correct ways of inferring the best explanation of some phenomena. Yet, these ways of thinking are not usually understood be logical. So, we might ask, what makes a correct way thinking specifically logical?

V. Logical Norms

Here we might take a cue from Frege who was of the opinion that logic is normative and

its norms are a subset of the norms of thought in general. Frege also had a particular

view about where these norms come from. Norms of thought, according to Frege, are

normative or prescriptive laws that arise from what we might call 'descriptive' laws. For

instance, he said:

> The word 'law' is used in two senses. When we speak of
>
> moral . . . laws we mean prescriptions, which ought to be obeyed
>
> but with which actual occurrences are not always in conformity.
>
> Laws of nature are general features of what happens in nature and
>
> occurrences in nature are always in accordance with them. It is
>
> rather in this sense that I speak of laws of truth. Here of course it
>
> is not a matter of what happens but of what is.[6] (1918 p. 58)

> Any law asserting what is can be conceived as prescribing that
>
> one ought to think in conformity with it, and thus in that sense a
>
> law of thought. This holds for the laws of geometry and physics
>
> no less than for the laws of logic. (1893 p. xv)

So according to Frege a descriptive law gives us a prescription or norm for

thought. When one is thinking about physics, for instance, the laws of physics give one

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[6] Following Macfarlane (2002) I take 'laws of truth' here to be equivalent to 'laws of logic'.

a norm according to which one ought to reason. For instance, if one knew that a certain car was going a certain speed and had a certain weight one could calculate, if one knew the laws of physics, the force which would be produced if it struck a stationary object like a wall. According to Frege then, one ought to reason in accordance with these physical laws when addressing questions from the relevant realm or inquiry.

Logical norms will not be exactly like this, but we might think of them, so as not to beg the question against the pluralist, as given to us by the facts regarding whether some conclusion is true in all 'cases' where the premises are true. Let us call these 'the case facts'.[7] Thus a logic must reflect in its norms the facts regarding what is true in the various cases, e.g. possible worlds or Tarskian Models. Reflecting the case facts is what makes norms logical.

It may be pertinent at this point to say that even though I've been talking in terms of logical norms the view I've been describing is compatible with many different views of normativity. We could think of the norms of logic as merely those rules one must follow to count as doing logic. They need not be thought of as norms that genuinely obligate us to think in any particular way, although they may be thought of that way as well.

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[7] I don't intend the sense in which I mean 'fact' here to commit us to realism about the 'facts' regarding cases, at least not in the sense where this contrasts with anti-realism. Most anti-realists think that an entity from whatever domain they are anti-realists about is dependent for its existence on someone's awareness or on some kind of epistemic constraint. But once this criterion, whatever it is, has been met presumably we could say, in some sense or other, that a fact has been established. If so even an anti-realist could recognize 'case fact' here whatever they turn out to be.

It's also important to make one clarification before going on to compare the criterion of closeness to the objective logic with the moral and epistemic criteria of correctness. The question of whether the correct formal logic by the moral or epistemic criteria is the logic that reflects the case facts might seem to make little sense if something is only a formal logic if it reflects the case facts. But remember that formal logics are logics designed to try and capture intuitively correct arguments. It could be a formal logic declares some of those arguments, which reflect the case facts valid but not all, or it may declare some that are invalid valid. Thus a formal logic may be more or less reflective of the case facts.

But the flip side of the coin is, as per the examples above, that if we were to ask which logic is correct meaning correctness to be understood in terms of a moral, or epistemic criteria we may be enjoined to choose a formal logic, which does not completely reflect the case facts. We may even choose one, which is farther from the facts than other logics available to us.

VI. Forms of Pluralism

But now that we have made the above points clear we can make some distinctions between several weaker and stronger forms of logical pluralism. One form of pluralism, which I will call 'weak pluralism', is the fairly innocuous view that several of the formal logics currently available have consequence relations that reflect some of the case facts from the one single monolithic set of cases facts, to some degree or other. A formal logic

will reflect the case facts to a greater or lesser degree in the following way. If the number of objectively valid arguments is **x** and the number of these arguments declared valid by logic L1 is **y** and the number of invalid arguments declared valid by L1 is **z** then L1 comes close to the perfectly reflecting the case facts insofar as **y** – **z** approaches **x**. There will be stronger and weaker forms of weak pluralism depending on how close these numbers come and for how many different logics.

A stronger form, but still quite weak comparatively, might maintain that there are two or more logics that tie for best representing the facts but are still distinct. Two logics can tie and yet be distinct if they declare different sets of arguments valid but the *number* of arguments they declare valid is the same. I'll call this 'moderate pluralism'. There could be many different degrees of moderate pluralism, as well, depending on the degree of overlap in the set of arguments the logics in question declare valid and the degree to which the above specified **y** – **z** approached **x** for each of the logics.

A still stronger, but slightly different view, might consider some more sophisticated views of moral or epistemic criteria for correctness. On this view there are decisive normative considerations for choosing several logics but these norms leave us in a moral or epistemic dilemma with no one logic winning out. This might be a purely moral or purely epistemic dilemma or some combination of or conflict between either of these. On this view which logic we ought to choose depends on something other than the degree to which it represents the case facts. We can call this, 'dilemma pluralism'. And there could be different degrees or levels of pluralism here too depending on how many

logics we are left in a dilemma about and perhaps how many different types of norms are in conflict.

While I think this kind of pluralism is conceptually possible it's difficult to come up with examples that are not wildly implausible. One example might go as follows: Suppose knowledge has value in itself as do certain states of affairs like x's-being-pleased-at-y. Suppose, also that I have an obligation to maximize these values. Unbeknownst to me, I will die in about two hours and thus the beliefs I form over the next two hours will have little effect on the world down the road.

I have been studying the properties of several logics trying to decide which one is correct. I am, subconsciously, drawn to relevance logic because it would make me feel good to be a rebellious maverick philosopher. But in fact some other logic has many more genuinely valid inferences. If I believe relevance logic for the next two hours I will increase my pleasure to a certain degree. If I believe the other logic I will increase my knowledge to a certain degree.

Suppose there is an objective scale on which to compare these two goods and they are equal or if you prefer suppose that they can be judged to be within a certain range of each other but which one is more valuable is not fully determinate and thus my obligation to believe one does not outweigh my obligation to believe the other. This might be a case of dilemma pluralism.

Of course, it relies on the idea that 'ought' does not imply 'can'. If 'ought' did imply 'can' then the fact that I can't believe both at the same time would mean that I could not possibly have both obligations at the same time. If this were the case then either I would not in fact be obligated to believe A and obligated to believe B but obligated to believe A or B, in which case the above situation would not give rise to a dilemma.[8]

A fourth and somewhat odd view would be that i) there are decisive normative considerations according to which we ought not take into account closeness to the case facts when choosing a formal logic, ii) there are at least two to choose from. We can call this, 'rejectionist pluralism'.

For a somewhat artificial example of this kind, suppose that there is supper intelligent but evil robot from the future. The robot captures and is threatening to kill some of my friends in a very short time if I do not solve some very complex logical problems for him. Suppose I have good evidence from my years of study that relevance logic is the logic closest to objective logic and this evidence is not misleading. However, the robot demands that I work in intuitionistic logic. On the supposition that saving my friends is morally obligatory for me I ought morally to do as the robot demands. Thus I have a decisive moral obligation to ignore the issue of closeness to the facts.

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[8] This presumes that I can choose to believe one of the logics or the other is correct, which may not be true. I may not have much control over what I in fact believe.

Weak pluralism is, I think, true. Moderate pluralism may be true and if it is not now it may be true at some point in the future as various formal logics develop, because it is always possible that two logics will declare the same number of objectively valid arguments valid. Whether one thinks dilemma pluralism is even possible depends on ones view of epistemic and moral norms. And rejectionist pluralism seems to be possible in any situation where we have a limited time frame.

Each of the first four views is a pluralist view about formal logics, a final view is that the case facts themselves give us several different sets of norms. So this is a view that maintains that there are, in fact, at least two objective logics. Call this 'strong pluralism'.[9] Someone might have such a view either because she thinks that one set of case facts somehow gives rise to several distinct sets of norms or because she thinks that there are several distinct sets of case facts. If one were to hold a strong pluralist view to the effect that there is more than one objective logic one might then go on to hold any of the above weaker forms of pluralism with regard to formal logics.[10]

Arguably Beall and Restall's view most closely resembles the last option. This is most apparent when we remember that their reason for endorsing pluralism as explicitly stated is that there are four distinct types of 'cases' which claims may be true in. Each of these gives us a distinct set of logical norms. Given that we defined 'case facts' as the

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[9] It may be worth pointing out that none of these forms of pluralism is mutually exclusive. One could be a weak and strong pluralist at the same time. In fact as far as I can see one could hold all or any combination of the views at the same time.

[10] If the distinct sets of objective norms, which arise, for either reason, could conflict then one might hold a form of dilemma pluralism with regard to objective logics too. On the other hand it could be that when any two norms conflict we are permitted to follow either one. This will almost certainly be the case if we think ought implies can for the sort of normativity involved here since we can't follow both at the same time.

facts regarding what's true in the various cases, Beall and Restall fall into the 'several distinct types of case facts' branch of strong pluralism.

VII. Recapture

There are several questions that arise, however, over the pluralist way of understanding validity. First, it is apparent that each of these ways does indeed give rise to a different set of arguments, which it declares to be valid. But in the same way that the overall consequence relation encompasses the more specific relations between the premises and conclusions of various arguments in a single logic, why can it not be the case that there is a yet more general logical relation which declares all the arguments valid which each of Beall and Restall's different senses of 'valid' apply to?

To be a bit more technical there is a phenomenon known as *logical recapture*. Roughly, what this means is that for some restricted domain of discourse one logic will declare all and only those arguments valid which some other logic always declares valid.

More precisely, we can represent a logic L, with wff formlas defined in a standard recursive way, as a couple $<W_i, V_i>$, where W_i is the set of wffs and V_i is the set of valid inferences in L.[11] The set of valid inferences will be a subset of the set of sequents of W_i, where a sequent is a pair of sets of wffs (a set of premises and a conclusion or set of conclusions). "Equivalence" of two logics then, "consists in a one-to-one

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[11] In this account of recapture I follow Aberdein (2001) and Read and Aberdein (2009).

correspondence between equivalence classes of the wffs of the systems which preserves the partitions of the classes of inferences into valid and invalid subclasses."(Read & Aberdein (2009) p. 16) Once we have these notions in place we can define a few relations that might hold between two logics.

Proper Subsystem: L1 is a proper subsystem of L2 iff L1 and L2 are inequivalent, W_1 is a proper subset of W2 and V_1 contains precisely those elements of V_2 which contain only elements of W1.

Recapture: L1 recaptures L2 iff there is proper subsystem of L1, L3, which is defined in terms of a constraint on W1 finitely expressible in L1 and which is equivalent to L2. If L2 is classical logic then L1 is a classical recapture logic. (ibid.)

As an example of what is meant by a finitely expressible constraint we can point to the case where the law of excluded middle is added as an axiom to intuitionistic logic.[12] When we do this the resulting system declares all and only classically valid arguments valid. In other words intuitionistic logic is a classical recapture logic.

When one adds *excluded middle* as an axiom the result is that from the classicists point of view all sentences must, appropriately in his eyes, be true or false. But if one persisted in ones belief as an intuitionist that some sentences are neither true nor false one would view the inclusion of *excluded middle* as making it improper to express sentences,

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[12] Aberdein (2001) p. 3

which are neither true nor false in that logic. Affirming that they were either true or false would be, by the intuitionist's lights, false and denying both would result in a contradiction.

But before we can use the notion of *recapture* in argument against the pluralist directly there is one technical issue that deserves discussion. In laying out the different kinds of validity the logical pluralist recognizes I have been talking in terms of arguments, whereas the account of *recapture* just outlined uses the notion of sequents. That is the logical pluralist says that when we evaluate logical arguments taking different substituends for 'cases' in the definition of validity we find that different *arguments* are declared valid. Whereas what we have said with the definition of *recapture* is that insofar as some expression of a restriction on the logic can be expressed different sets of *sequents* will be valid.

So we ca not evaluate what *recapture* tells us until we know how sequents relate to arguments. For Beall and Restall what an argument is, is left somewhat vague. They want to leave it open whether an argument is made up of propositions, mental representations, regimented sentences, statements or some other more elaborate items. But arguments are not uninterpreted sentences in a formal language. On the other hand a sequent, remember, is an ordered pair of a set of wffs; a set of premises and a conclusion.[13] Wffs have no meaning in themselves until they are provided with an interpretation.

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[13] If the logic is a multiple conclusion logic there can be a set of conclusions.

An example of a sequent then is $<\{a{\to}b, a\},\{b\}>$. This sequent is of the form Modus Ponens. But then so is $<\{c{\to}b, c\},\{b\}>$, which is distinct. So sequents allow more fine-grained differentiation than schemas. But the sentences "If the cat is on the mat then he wants to be let back in. The cat is on the mat. So he wants to be let back in" could be translated into a formal language and end up as either of the sequents above.

In order to make sure that all arguments have a single representation in sequents we can stipulate that our formal language contains a distinct specific predicate symbol for every legitimate predicate of the language we are translating from and a distinct variable for every name in that language. Once we have such an arrangement there will be multiple ways to establish a one to one relationship between sentences in natural language and sentences in the formal language. But as long as we stick to one translation scheme, always translating the same natural language predicate with the same formal language predicate symbol we can decide whether a restriction on the formal language allows us to preserve the partitioning of arguments into valid and invalid for the each of the forms of validity the pluralists recognizes.

Above I said that the formal language should contain a predicate symbol for each legitimate predicate of natural language. I included the qualification that the predicates must be legitimate because we may need to avoid having a truth predicate and other predicates that express semantic concepts. Doing so would allow us avoid the semantic paradoxes like the various forms of the liar, e.g. "This sentence is false". This will slightly restrict the arguments that can be expressed in the formal language. However,

this is not a special problem for my view but a general problem that any logician will

have to deal with when translating from natural language into formal logic.[14]

In any case, if this account is successful then *recapture* makes the logical

pluralist's job much more difficult. Suppose for instance the pluralist claims that there

are two logics L1 and L2, which yeild valid argument sets V1 and V2 respectively. If V1

and V2 can be recaptured inside some single logic then, at the very least, there is an

alternative explanation from the pluralist's of the apparent validity of the arguments. So

the pluralist must either prove that her logics cannot be recaptured in this way or argue

that her explanation of the validity of V1 and V2 is more plausible than the explanation

of those who embrace the single logic, which recaptures them.

On the other hand, she may also take an even more aggressive route and question

whether *recapture* is relevant at all. She may question whether the sense in which the

logic is recaptured is the same as the original sense in which it was first proposed. If a set

of sequents is declared valid in one logic but only valid for a restricted set of formulas in

another, one might feel that this difference in the circumstances under which the validity

holds is enough to raise questions about whether 'recapturing' a logic is anything but an

interesting technical result. For the pluralist the logics L1 and L2 are valid because the

cases their component sentences are true in differ, whereas for the monist who recaptures

L1 and L2 the difference is linguistic. Perhaps this difference is enough to lend prima

facie legitimacy to one who questions the relevance of *recapture*.

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[14] Anyone familiar with Tarski's work on truth will likely be familiar with the problem here. See Tarski (1944).

It may be that whether one is likely to concede that *recapture* shows anything significant will depend on one's extra-logical presuppositions in various subject matters. It will be important to examine issues about what quantifying over certain kinds of objects commits us to, and, keeping this in mind, whether it's correct to restrict the language or expand it in various ways.

For instance, consider the case of adding *excluded middle* as an axiom to intuitionistic logic. Surely part of the motivation for formulating intuitionistic logic was that its designers felt certain that there were some expressible propositions that were neither true nor false. So, presumably they would object on independent grounds that this kind of *recapture* is in fact irrelevant because it makes illicit extra-logical commitments; commitments to the truth or falsity of any given formula.

So deciding this issue will require looking into the background theories and assumptions we bring with us, fail to bring with us, or perhaps even consider, when doing logic. It might require us to examine what the objects quantified over *are*, metaphysically speaking, the nature of the representations one thinks a logic has, whether one thinks results in logic can commit us ontologically, etc., etc. For instance, are the elements of the domain abstract objects? If so do we have a theory of abstracta which assumes that there will always be enough objects to supply an interpretation for all the expressions of a language?[15] Is this theory of abstracta independently justified? Similar questions may be multiplied nearly at will for the other issues alluded to. In fact we should go on to explore these issues for all the 'cases facts'.

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[15] As is well known now this is a question raised by John Etchemendy in his (1990)

Even after we have properly investigated all these issues however, we may still not be able to prove that for any two formal systems proposed as distinct logics they can always be recaptured by some third logic, or that there are at least two logics for which this is not possible. For all we know, there may be no finite way to prove such a thing. So the degree to which this will be a problem for the pluralist in particular will perhaps have to await further clarification of background issues.

VIII. Extra-Logical Connections

Nevertheless we needn't go on to try to prove things about *recapture* if we begin to explore some of the background issues. And as with the case of the intuitionist and *excluded middle* above there may be independent argument for or against logical commitments that will help us decide on the plausibility logical theses. This is important because if one wishes to make an argument for the correctness of a certain logic, or certain logics, one must make them in a particular logic. Ideally one would make one's case using only those arguments that both oneself and one's opponent accept as valid. But we should first remember that there is a distinction between those arguments that a logic always declares valid, in this case I will say it declares the argument universally valid, and those it declares valid only over a restricted domain, which I will say it declares restrictedly valid.

It will not always be the case that any two logics declare some of the same arguments universally valid. And it may not always be the case either that they can find

some restricted language for which they declare some of the same arguments restrictedly valid. But failing to find cases of restricted overlap, as we will call it, is less likely. Assuming, however, that we could find an overlap in the arguments we and our opponent consider valid each could then recognize the others reasoning as valid and thus carry out a meaningful debate.

But even if we could not argue directly using arguments we both recognize as valid this does not mean we could not make some form of prima facie argument. Our intuitionist above might, for instance, make an appeal to something other than a deductive argument, which he thinks justifies his belief in exceptions to *excluded middle*. He and his interlocutor may both accept a form of reliabilism with regard to epistemic justification. If so he may appeal to a certain beliefs his opponent is likely regard as reliably produced, if these beliefs might help him to make his case. Alternatively, he may feel some of his beliefs are justified by being more explanatory of other justified beliefs. In any case, the point is merely that given the somewhat circular nature of arguing *for* a logic *in* a logic it will be important to pay attention to which beliefs or propositions we and our opponents are more justified in accepting and whether any of these may help us make the case for our position.

Still, while it is important to recognize the possibility of proceeding in this way, it's also important to keep in mind an earlier distinction I made regarding the criteria for 'correctness'. Unless one is an epistemic infallibilist, it will always, in principle, be possible that the logic we are justified in choosing is not, in fact, the logic whose norms

or laws most directly reflect the case facts.[16] In other words, there may be a difference between the correct logic to choose by epistemic criteria and the logic closest to reflecting objective logic.

One can, of course, hope that as one collects more evidence one will move closer towards recognizing the facts for what they are. But before I go on to address 'case facts' and what, in fact, they are supposed to be, it's worth looking at one other possibly contentious point.

I have said that we might become committed to certain extra-logical theses by way of our logical ones and vice versa, and I have contended that this is advantageous because it gives us a potentially wider field of justified beliefs against which to measure our logical theories. But in what sense does a logical thesis commit us to an extra-logical thesis? If by 'commit' I mean entails then clearly this too will be contingent on which logic is correct. If this were the case it would do no good to try to constrain our discussion of the correct logic by bringing in extra-logical theses. There are two important responses that can be made here.

First, it seems intuitively plausible that some very basic argument forms will be highly justified for us without argument. I happen to think that arguments conforming to the Modus Ponens schema have something like this status. If they do then we can be

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[16] Though it's true, of course, that if we could be infallibly justified then there would be no gap between the logic we're justified in accepting and the logic which fits the facts it seems absurd to hope that we have access to an infallible intuition or 'rational insight' regarding logic at this point. The only way to argue for such a view now would be to maintain that all those who disagree with one's own logic, pluralist or not, are simply being insincere or deceiving themselves.

justified in using arguments of this form in our reasoning.[17] A logician who did not

accept Modus Ponens would need some pretty strong evidence to convince us that these

are invalid. Whether one thinks one holds some externalist-reliabilists account of

justification, or an internalist account, or something else entirely does not matter for

present purposes.[18] And in addition, as we have pointed out, one might be more justified

in accepting some metaphysical or meaning theoretic truths than one is in certain logical

truths.

Here we might consider again the intuitionist as an example. In order to make it

more clear what's going on here it's worth giving a short background story. The main

feature by which intuitionistic logic differs from classical is by its denial of *excluded*

middle as a logical law.[19] Brouwer, who first formulated the view, held a theory

according to which sentences or thoughts about things that do not exist are neither true

nor false. And according to Brouwer since mathematics is merely an "activity of exact

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[17] This is not to say that if we have some justification for thinking that arguments of the Modus Ponens form are valid then we are automatically justified in using the form on every occasion. I'm merely claiming that we could be justified in using it. What is involved in being justified in using a certain form of argument is a substantive question. Does it require that we are explicitly aware that we are using such an argument form. I think most epistemologists would deny this on the basis of the fact that this could lead to a regress on an analogy with Lewis Carrols tortoise where each time we give some reason for believing our argument valid we have to justify the reasoning we used argue that it was justified. But what exactly is required is an open question.

[18] Again, whether or not these justified arguments actually conform to the 'case facts' is a different question, involving us again in the distinction between the logic we are, if were not epistemic infallibilist, justified in accepting and the true one.

[19] The law of excluded middle is not always understood the same way. Sometimes it is treated as the same as the law of bivalence and at other times as a separate principle. When they are separated it is often said that the law of excluded middle is 'every proposition is either true or untrue' while the law of bivalence is the stronger claim 'every proposition is either true or false'. Geach, P. and Bednarowski, W. F. (1956). In the forms above it's not really clear whether all intuitionists want to deny *excluded middle* or bivalence. Most likely intuitionists will differ here and many have probably not explicitly considered the question. I see no reason to think it would affect the point above either way so I won't make an explicit distinction between the two here. However, it should be noted here that intuitionists cannot deny instances of *excluded middle* in the object language in which they are working. This is because they accept the Demorgan's laws. Thus denying an instance of *excluded middle* in intuitionistic logic would be convertible from ¬(p ∨ ¬p) to p ∧ ¬p which is a contradiction. (Thanks to Joe Salerno for pointing this out to me) But notice that the 'instance' of *excluded middle* in question is written as 'p ∨ ¬p' in the object language. This is not obviously derivable from the meta-linguistic version above. The reason it's not is that it is assumed that things like negation (un- in 'untrue') in the meta-language function like the constants of classical logic, whereas if we accept the denial of *excluded middle* then the negation in the object language cannot be classical. Specifically, intuitionist negation doesn't obey the law of double negation elimination. So the '¬' in the meta-linguistic '¬(p ∨ ¬p)' is different from the '¬' in the objection version.

thinking"[20] a statement about the truth of some mathematical principle will be true when the 'exact thinking' that constitutes a proof has been performed. So until such time there is nothing to make any statements about the formula true.

So for instance if no one has proved Goldbach's conjecture then there is no determinate mathematical fact for the statement "Goldbach's conjecture is true" to be about. Thus such a statement is neither true nor false. So what Brouwer takes to be a particular metaphysical fact, in this case the mind-dependence of mathematical truths, and a particular view about the truth conditions of propositions has an effect on Brouwer's choice of logic, insofar as his views on these matters provided the motivation for him to give up the law of excluded middle.

Someone may object to the characterization above, that committing ourselves to the view that some sentences are neither true nor false is the same as a rejection of *excluded middle*. If this were true the above example would merely cloak a straightforward rejection of that principle. But notice that one could hold the following principle, which I will call the 'anti-Vulcan principle' after the famous non-existent planet: if any sentence were to attempt to predicate something of its subject term but that term turned out to be non-referring the sentence would be meaningless. One might hold this whether or not such sentences actually occur or are even possible. One could then go on to hold that if meaningless sentences are to be given a valuation in a formal system they should be evaluated neither true nor false. Call this the 'valuation principle'.

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[20] Van Atten (2005)

Trivially neither the first nor the second principle is equivalent to a denial of *excluded middle* or a statement of its negation in their current forms. The first is a subjunctive conditional, whereas a denial of *excluded middle* though it can be stated as a conditional, is usually a disjunction. In addition even if it were stated as a conditional it would be an indicative conditional, and in any case is about truth and untruth whereas the anti-Vulcan principle is about meaningfulness. We could try converting the anti-Vulcan principle into a disjunction thus, "Either it would not be the case that some sentences purporting to predicate something of its subject would turn out to have a non-refering subject term or that sentence would not be meaningful." This makes for a somewhat cumbersome formulation but in a possible worlds semantics it could be understood as saying that in any possible world either there will not be some sentences having the properties in question or that sentences will be meaningless. The law of excluded middle on the other hand could be formulated thus "For any proposition p either p is true or p is untrue." This differs from the anti-Vulcan principle in that it makes no reference to meaningfulness and it has no subjunctive element. The valuation principle on the other hand is about how a non-meaningful sentence is to be given an alethic value if it is given one. It could be interpreted as follows, "For any proposition p if p is meaningless then for any truth values T, F, i, etc. it is not the case that $v(p)=T$ or $v(p)=F$." Here $v(p)$ is a valuation of p assigning to it a truth value, T means 'it is true that', F means 'it is false that' and i means 'it is indeterminate whether'. So, the valuation principle is not equivalent to a denial of *excluded middle* either.

More importantly we might think that conjoining the two sentences would amount to a denial of *excluded middle*. And surely any logic that is to be acceptable, we might argue, would allow that if A and B are both true then A ∧ B is true.[21] But given the subjunctive element of the first the conjunction of the two would retain a subjunctive element and thus would still not be equivalent to a denial of *excluded middle* at least so long as its not actually the case that there are sentences with non-referring subjects. When a subjunctive is evaluated at the actual world is it for all intents and purposes evaluated like its corresponding indicative. But even the conjoined principles evaluated at the actual world would retain the reference to meaningfulness, which *excluded middle* lacks. So it would still not be strictly equivalent to a denial of *excluded middle*.

In any case a person who held this view would be committed to rejecting the law of excluded middle because she held a certain thesis about the meaningfulness of language only if a certain kind of reference failure were not only possible but, in fact, actual. So this is not just a straightforward rejection of *excluded middle* on its own merits but a principled rejection on the basis of extra-logical thesis which the intuitionist, at least, takes to be independently plausible.

So we can see how holding some particular extra-logical theses can affect one's logical commitments. So long as one is genuinely more justified in holding them than in the logical theses they are in conflict with there is nothing problematic about this. But

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[21] Of course it's been shown that everything that can be done in the standard logic with the connectives ¬,∧,∨,→, and ↔ can be reproduced while restricting the language to contain only certain pairs of these, or only | (nand). But in these cases sentences truth functionally equivalent to conjunctions can be expressed. So these are not *really* doing without the ability to conjoin. There are logics, however, for which conjunction introduction is not valid, see Berto (2007) § 2.2. In any case we needn't show that no logic can deny conjunction intro, but merely that we are prima facie justified in accepting it.

now an interesting question arises. What kind of extra-logical commitments does logical

pluralism itself have?

IX. Case Facts

More specifically we might ask, what exactly a case is. Addressing this question

Beall and Restall respond that "Cases, whatever else they are, are 'things' in which

claims may be true." (2006 p. 89) Obviously we need a little more clarity on just what

they take this to mean. First, let us consider what Beall and Restall think about truth.

There are many theories of truth. There is the correspondence theory, the

coherence theory, the identity theory etc. One can be a relativist about truth or non-

relativist. Beall and Restall never advance a highly specific thesis about truth. But there

are a few important questions that pluralism about logic raises right away. If there is

more than one consequence relation, then it seems plausible that, given some set of facts,

different things will be entailed by these facts depending on which consequence relation

is being considered. This possibility immediately raises the question of whether a logical

pluralist is committed to a form of relativism about truth. Relativism about truth is the

view that what is true may differ relative to either a particular person, or a culture or

something else.

Now it's hard to see how a form of relativism would work here but presumably

since the problem was that different truths might be entailed on different consequence

relations, this would involve the consequence relation itself holding relative to some further fact. Fortunately Beall and Restall save us from having to work out in detail whether this even makes sense. They explicitly reject relativism about consequence, saying, "We are not *relativists* about logical consequence, or about logic as such. We do not take *logical* consequence to be relative to languages, communities of inquiry, contexts, or anything else." (2006 p. 88; emphasis in original)

On the other hand they explicitly endorse a form of *pluralism* about certain truths, namely logical truths. As they put it, "Given a logical consequence relation defined on the class of cases$_x$, the logical$_x$ truths are those that are true in all cases$_x$."(2006 p. 100) The obvious intent here is to say that there is more than one set of logical truths given that there is more than one set of cases. And it's clear that if the sets of logical truths differ then there must be different non-logical truths that hold in different cases. But what does it mean to say that different things are true in different cases? Is something true in each kind of case in a way similar to the way things are true in possible worlds. Many philosophers think they understand what it might mean for something to be true in a possible world. But what would it mean for something entirely different to be true in another kind of 'case'?

Beall and Restall seem to take it for granted that if one can specify that some formulas are true in a possible world or a model then there are facts with respect which a logical consequence relation can be specified. They say, "By specifying 'truth conditions' for claims, you thereby specify cases." (2006 p. 89) They then go on to say

that it's a separate issue whether such cases will give one a consequence relation. This depends on whether the class of cases "affords an admissible instance of" (ibid) their definition of validity. The admissibility of an instance is, in turn, determined by whether the resulting candidate relation meets what they think are some of the widely agreed upon features of a logical consequence relation.

One of these features is necessity. As they put it "An instance of [the general definition of validity which one gets by specifying a kind of case] is *admissible* . . . if its judgments about consequence are necessary . . ." (2006 p. 35) Later they specify that this means that, "it must be necessary that if the premises are all true so is the conclusion." (2006 p. 40) In other words there must be some kind of necessary connection between the premises and the conclusion.

This condition is similar to a definition of validity, which is slightly different from Beall and Restall's version invoking cases. This alternate version, 'the impossibility definition', is arguably the more common definition as well. It goes as follows: An argument is valid iff it's impossible that its conclusion be false when its premises are true.

But as I pointed out above in the example involving "A is red. Therefore, A is colored", (the 'red-color example') Tarskian models would allow the conclusion to be false while the premise is true, and vice versa, even though the possible worlds interpretation would not. All that is needed for the formula $F(a)$ to be true is for the

object assigned to 'a' to be in the set of objects assigned to 'F' and all that is needed for the falsehood of G(a) is for a to fail to be in the set assigned to G. And since we are required to go through every different combination of assignments for 'F', 'a' and 'G', of course cases will arise where the object assigned to 'a' does not belong to the set assigned to 'G' even when it belongs to F. Thus rendering 'F(a) |–G(a)' invalid. Thus models clearly contain impossibilities.

So the logical pluralist is committed to quantifying over impossibilities. And, given the above account of the truth-conditions, is committed to the truth of impossible sentences like "A is red and A is not colored". Is this a problem? Well if we continue to inter-define 'impossibility', 'necessity', and 'possibility' it appears to directly contradict the necessity constraint. But, Beall and Restall avoid conflict here because on their view necessity also comes in different kinds. This is because they define necessary truth as follows: "A claim is necessarily$_x$ true if and only if it is true in every case$_x$." (2006, p. 26)

But is this plausible? We have already seen that any time you specify any set of truth conditions you have specified a case and if you have done so you have in turn specified a new sense of necessary truth, truth in all the cases you just specified. But it does not seem that merely specifying a new set of truth conditions is sufficient to create some new necessary truths. Intuitively there is a fact of the matter as to what is possible and what is necessary, and thus what is possibly true and necessarily true.

Now there are many different philosophical accounts of modality. Most contemporary philosophers, however, give accounts of modality in terms of possible worlds. And there are in turn many different accounts of possible worlds. One common division has it that one may be a possible worlds realist, fictionalist, or an ersatzist. The realist takes possible worlds to be concrete space-time regions just as real as the actual world. The fictionalist takes them to be, naturally, fictions of some kind similar to literary works of fiction and the ersatzists take them to be abstract objects of some kind. But these are merely different views of what possible worlds are. It's a separate matter whether one who holds any of these views wants to go on to offer a reductive account of possibility or necessity in terms of the theory of possible worlds on offer. Many theorists take modality to be primitive. That is, they think that what is possible and what is necessary must determine whether our account of possible worlds is plausible and not that our account of possible worlds is to determine what we see as possible and necessary.

There are, however, views that attempt to reduce modality itself. For instance there is a distinction between strong and timid modal fictionalism. Timid modal fictionalism simply tries to account for talk of possible worlds in terms of truth in fiction. On such a view if I say that in some possible world Kennedy was not assassinated this is not interpreted in such a way that it commits me to the actual existence of possible worlds. Strictly speaking it is false that Kennedy was not assassinated at some possible world. But it is said to be true in the fiction of possible worlds in the same way it is true in a certain fiction that Spiderman is Peter Parker. But the timid fictionalist nevertheless

holds that it is true that it is possible that Kennedy was not assassinated. The purpose is merely to avoid commitment to the actual existence of possible worlds themselves. Strong modal fictionalism, on the other hand, tries to reduce modality itself to truth in a fiction.[22] According to this theory then when I say that Kennedy was not assassinated in some possible world, not only is it strictly false that there is such a possible world but it's false that it is possible that Kennedy was not assassinated though it's true in the fiction of possible worlds.

So then is pluralist's view of cases analogous to the strong fictionalist view of modality or is it more similar to the timid fictionalist? According to Beall and Restall, "[b]y specifying 'truth conditions' for claims, you thereby specify cases." (2006 p. 89) This does sound like something analogous to modal fictionalism. But if we also recall that a "necessary$_x$" truth is something, which is, "true if and only if it is true in every case$_x$" (2006, p. 26) it begins to sound like not just something analogous to strong fictionalism but is in fact a version of strong fictionalism. If the only necessary condition for a new type of case is that it be specified and if each new specification also gives rise to a new type of necessity this certainly sounds something like a fictionalist view.

If this is true then some of the main objections to strong fictionalism will be applicable here as well. For instance, what exactly is sufficient to specify a case on their view? Is there a new type of case, the type case$_x$, only when a set of truth conditions has been specified or is it enough that the truth conditions are in principle specifiable? If the former, then whether or not there are such cases depends on someone actually specifying

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[22] These two versions of fictionalism are distinguished in Rosen (1990)

them and logical consequence relations will depend on this as well. Thus, despite their denial Beall and Restall would become relativists about consequence since logical consequence$_x$ will depend on the specification of cases of the type case$_x$. And the specification of this type of case in turn depends on someone's specification of a set of truth-conditions. Furthermore, since they see claims as being 'true in cases' the existence of new cases would presumably give rise to new truths. Thus they would presumably become relativists about truth as well.

On the other hand if they took the view that truth conditions merely need to be specifiable in order to give rise to cases they would be committed to a primitive form of modality. This is not in itself problematic. It's not straightforwardly incoherent to argue that, in fact, there is one primitive form of modality on which the existence of cases is dependent.

And despite the fact that they appeared to be committed to a form of strong fictionalism on the basis of the quotes offered above there is in fact evidence to the effect that they do indeed recognize a primitive form of modality. To see this remember that even though truth conditions are specified recursively, as I pointed out above, this only specifies that certain formulas, those which result from joining two atomic formulas with a connective, are true relative to the atomic ones. So what determines the assignments of truth values to atomic formulas? Thinking back to the red-colored example, remember that they assumed that the argument would fail to be true in all possible worlds. They simply offered this example assuming we would agree that this is intuitively plausible.

The justification for this assumption is presumably that we do think there is a primitive sense of 'possible' in which it is not possible that something is red and not colored. So Beall and Restall do appear to assume a kind of primitive modality.

Still they also said that when any specification of truth conditions specifies cases and also specifies a kind of necessity. Recognizing a form of primitive modality allows them to maintain that the other cases exist in so far as they are specifiable, or *possibly* specified, in the primitive sense of 'possible'. So it begins to look as if Beall and Restall are timid fictionalists after all. But remember that the timid fictionalist merely wants to provide a reduction of possible worlds. She takes modality to be primitive.

But if the view above is really Beall and Restall's it is slightly different. On this view *one form* of modality is used to specify *other forms* of modality not just possible worlds. To help bring out the oddness of this kind of view recall that we defined objective logic as whatever makes genuinely valid arguments valid and we defined the logical norms to be those norms that arise from the case facts. Suppose we go on to say that the objective logic will correspond with the logical norms arising from these case facts. 'Case facts' remember was merely short for the facts about what is true in some set of cases. If the cases, with the exception of the possible worlds, turn out to exist in so far as they are possibly specified then it looks as if Beall and Restall are not merely fictionalist about some forms of modality but about some kinds of logical consequence as well.

This would be a very surprising result. But charity requires that we look for an alternative interpretation. Perhaps when Beall and Restall said that "[b]y specifying 'truth conditions' for claims, you thereby specify cases." (2006 p. 89) and then went on to say that for each new kind of case there was a new kind of necessity they did not mean that this specification of cases was a creative act. Perhaps they merely meant that there will be some genuine primitive sense of necessity which corresponds to any set of truth conditions one can specify. I am not sure this is any more plausible than the first view.

It would be difficult to estimate the number of distinct possible ways to specify a set of truth conditions. The number would partly depend on how many distinct sets of logical constants, such as ∧ or ∨, we felt we recognize. For instance, some logics might only have one constant, others will have three or four. Would we allow logics that have hundreds or infinite numbers of constants? If so the number of distinct possible sets of truth-conditions would be equally infinite.

But supposing we restrict ourselves to the standard four constants. Technically one can make the truth of a sentence vary in any way relative to other sentences. One could, for instance, declare that all sentences with even numbers of conjunction symbols were simply false or that sentences with predicates F and G behave differently under disjunction than sentences without those predicates. So it appears that the number of distinct sets of truth conditions is likely to be infinite. At the very least it would be an un-parsimonious view of modality to think that for each distinct set of truth-conditions there is, in addition to metaphysical modality, an additional genuine form of modality in the

world corresponding to this new set of truth-conditions. In addition, as some of the examples above illustrate some of these possible truth-conditions are quite odd. A view according to which each of these was a genuine form of modality would require significant argument to support it.

I think this is the most pressing issue for the pluralist. If she denies a primitive form of modality she becomes a relativist. If she defines other forms of modality in terms of one primitive sense she risks being committed to fictionalism with regard to the other senses of modality and of logical consequence. But if she tries to remain committed to her view of cases as arising from any set of truth conditions and still recognize multiple primitive sense of modality, then she will end up with a wildly un-parsimonious class of distinct modalities. And at the very least any of these options will require much further argument to defend.

X. A Concession

On the other hand, suppose instead that we think the specification of truth conditions should be constrained by our intuitions about the modal facts. Still it must be admitted that it is common among modern philosophers to accept several different senses of necessity and corresponding senses of possibility. At the very least there is generally thought to be a distinction between what is metaphysically possible, what is epistemically possible, what is logically possible and perhaps there are also forms of conceptual possibility as well. So, if we think back to the definition of validity according to which

an argument is valid iff it's impossible for its conclusion to be false while its premises are true, might there not still be a form of logical pluralism? As long as we still inter-define impossibility as that which is not possible then each sense of 'possibility' seems to give rise to a legitimate sense of 'validity'.

I cannot go through all the different senses of 'possible' that have been suggested at one time or another in the space I have here. But, given that we are discussing logical pluralism it's probably not plausible to think there is an independent sense of logical possibility for us to work with, so I will not discuss this notion. In addition, the notions of epistemic and conceptual possibility may, arguably, be open to multiple different precisifications.[23] So I will restrict myself to some examples explicitly discussed by Beall and Restall. One example involves metaphysical possibility, one involves a notion of conceptual possibility, and finally I will discuss a third interpretation of validity, which uses a notion more akin to conceivability.

If we interpret 'possible' here to mean 'metaphysically possible' then the inference from 'A is red' to 'A is colored' is valid. But consider the corresponding conditional, "If A is red then A is colored". In order for this sentence to be false it would have to have a true antecedent and a false conclusion, which is obviously impossible. So on the view under discussion this would be a logical truth, given that it would be true on every metaphysical possibility.

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[23] See for instance Huemer (2007) and Orestes (2005).

Even worse, however, "x is water. Therefore, x is H2O" might also turn out, if Kripke is correct about rigid designation, to be valid and its corresponding conditional logically true.[24] This would threaten to make logic at least partially an a posteriori discipline. We would have to perform scientific investigations to discover some logical truths, namely those taking the form of theoretical identity statements like 'water is H2O'. This, I think, would be a very puzzling development. It has traditionally been thought that logic is an a priori discipline. That is it should be, in principle at least, possible for any one who thinks about the subject intently enough to discover the logical connections between statements and to formulate logical truths. If we were now to be told after centuries of logical endeavor that some logical truths can only be discovered empirically this would be a very surprising development indeed.

In fact this would raise questions about the whole purpose of the enterprise of logic. While it may be fine and good to acknowledge a class of arguments that are not possibly wrong in the way the red-colored and water-H_2O arguments are, what purpose would such arguments serve. Arguing from red to colored may seem pretty straightforward and most people would probably not blink at it. This would seem to be one of those truths, which we used to call analytic. But, it still seems open to question whether it should be counted as a *logical* truth. As Beall and Restall note here, "While there may be genuine discoveries of valid arguments that we had not previously recognized as such, it is another thing entirely to think that these discoveries require empirical investigation." (Beall & Restall (2005) § 2)

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[24] Beall & Restall (2005) § 2

As I see it both of these arguments are problematic because they do not provide argument forms that are repeated in further arguments, which we have not yet encountered. We can, of course, repeatedly analyze concepts. Analyzing the concept of red might allow us discover some necessary truths like 'red is a color' or 'all red things are colored'. But this is quite different from taking Modus Ponens, or more likely some more complicated argument scheme, and recognizing that two currently accepted premises have the forms of the premises of that argument schema. Once you are able to do this you have a method for making deductions. To put it differently once a person is taught to see that arguments are valid in virtue of formal features she can learn to formulate new arguments by recognizing what argument forms propositions she believes would fit into.

Arguably this was part of the purpose of formal logic when Aristotle first began trying to formulate it twenty-four hundred years ago. But I do not want to argue here that all of logic, in the discipline sense, has the purpose of cataloging valid arguments to be used in deduction. Nevertheless, I think this does give us a hint. Since the at least middle ages the difference between an argument like the red-colored argument that met the definition of valid and arguments that took more standard forms like Modus Ponens or Tollens has been recognized. The distinction was referred to as the formal/material consequence distinction.[25]

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[25] According to Catarina Dutilh-Novaes the distinction was first introduced in Ockham, Summa Logicae, III-3, cap. 1, 45–58 (p. 589). Cf. Dutilh-Novaes (2004) n. 16

The hint then is that, as many have argued, properly 'logical' consequence is consequence in virtue of form. And arguably this is what is missing so far in our definitions of 'validity'. The definition either needs to be supplemented with some further constraint involving formality or another interpretation or definition needs to be offered. Unfortunately the idea of adding a formality criterion is complicated by the fact that many different understandings of what makes something formal have been offered over the years by many of the most famous and respected philosophers and logicians. According to John Macfarlane there are, historically, at least three kinds of formality.[26] Some have argued that logic is formal insofar as it provides norms for thought as such, because, being formal, it is not appropriate merely to one subject matter or another. Others have thought logic is formal insofar as it abstracts from differences in objects, and still others insofar as it abstracts from semantic content. Unfortunately I can not take time to comment much on these matters here. I merely mean to point out that the issue is not without its own problems.

Stephen Read argues that we can resolve the issue by defining the distinction between formal and material consequence thus, "Every instance of a formally valid argument is valid. But not every instance of a formally invalid argument is invalid." (Read 2006 p. 5) The instances of a formally invalid argument, which are valid are materially valid. He takes this to have provided a pretty clear criterion of difference between the two classes of arguments and he thinks that logic should only be concerned with formal consequence.

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[26] Macfarlane (2000)

To be more specific it's clearly not the case that every argument of the form 'F(a)

|= G(a)' is valid. But given that the red-colored inference comes out valid on our most

recently explored definition of validity it is one instance of that form which is valid.   On

the other hand every argument of the form of Modus Ponens is valid.[27] But it might be

pointed out in response that this presupposes a particular notion of a logical schema.

Modus Ponens is an argument schema but, as Beall and Restall point out, so is the

following: "*x* is *y*'s mother's brother's son. Therefore, *x* is *y*'s cousin." (Beall & Restall

(2005) § 2)

Not only is this argument valid, but it appears that every instance of it will be

valid too. So in order for Read's idea to work we would need to put some restriction on

what would count as a schema. But we would need to do so in a principled way.

Part of the solution might come from a stipulation that everything must be

replaced by a schematic letter except for logical constants such as 'and', or 'or'. But this

faces the problem of specifying the nature of the logical constants. This is an issue

currently under a lot of discussion.[28] But as with the notion of formality above I do not

have time or space to discuss it here. Suffice it to note that how to delineate what is to

count as a logical constant is controversial as well. So an appeal to logical constants to

solve our difficulty with schematization will have its own problems.

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[27] Actually this has, not surprisingly, been challenged and not just by paraconsistentists. See Woodbridge, J &
Armour-Garb, B (2006)
[28] Cf. Sher (forthcoming) and Fefferman (1999)

But, in any case, we can see that interpreting the consequence relation to require truth preservation across merely metaphysical possibilities is prima facie problematic. So what about interpreting the kind of possibility at issue as conceptual possibility? This kind of possibility is arguably more suited to logic if we think the purpose of logic is, as I said above, to allow us to reason by applying argument forms repeatedly to new sets of premises as we encounter them, to expand our knowledge. It makes more sense because we need not rely on a posteriori knowledge of the world to find out about certain modal truths like 'water = H_2O'.[29] It's conceptually possible that 'water = H_2O' is false, so appealing to conceptual possibility prevents this from being a logical truth. This is certainly more in keeping with our intuitive notion of logical reasoning.

But Beall and Restall have an argument against appealing to conceptual possibility as well. They point out that with regard to the argument, "Peter is Greg's mother's brother's son. Therefore, Peter is Greg's cousin . . . [o]ne can understand that the conclusion follows from the premises, on the basis of one's understanding of the concepts involved." (Beall and Restall 2005 §2) Yet this too appears to be a strange candidate for logical truth. Again, it looks like what we need is a way to single out arguments, which are valid in virtue of their form. And again we will have to await an account of form before we can do so.

But there is a further way of understanding the kind of possibility at issue. As I pointed out above Tarskian models make use of impossibilities in evaluating arguments for validity. I also pointed out that there is a notion of impossibility involved in a very

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[29] That is, so long as we're not semantic externalists of a certain stripe.

common definition of validity. And it's commonly the case that impossibility, possibility and necessity are inter-defined. Often the necessary is defined as that which is true in every possible world, and the impossible as that which is true in no possible world. As noted Beall and Restall define 'necessity' as *truth in all cases*. Given all of this it seems possible to define a sense of 'possibility' according to which each case represents one such possibility. But this would certainly create a tension with the recognized impossibilities already noted. One way to resolve this would be to stop defining 'impossibility', 'possibility', and 'necessity' in terms of one another. Another way would be to use multiple distinct sense of each term. For instance, we might say that it is impossible$_1$ that something is red and colored. This sense of 'impossible' would be correlative to possible$_1$ and necessary$_1$. A new sense of 'necessity', necessity$_2$, would then be introduced. This would be the sense in which something true in all cases$_1$, even the impossible$_1$ cases, is necessary.

And in fact we saw above that the pluralist does say that necessary$_x$ truth is something, which is, "true if and only if it is true in every case$_x$." (2006, p. 26) So they at least pave the way for taking the second option on which there to be a different kind of necessity at work. But it's not clear what any corresponding sense 'possible' would be.

Beall and Restall interpret conceptual possibility in such a way that it's not conceptually possible that someone could be your mother brother's son and not your cousin. But it is at least conceived. So perhaps the notion of possibility were looking for is really conceivability. It's not clear, however, that we could count conceivability as

simply a wider form of possibility. If not then it would not, strictly speaking, fit into our definition of validity as it stands. Indeed some have argued that conceivability entails metaphysical possibility, so it's at least controversial whether conceivability could be a meaningfully separate sense of 'possibility'. But in the context of a debate over logic itself, it's also plausibly controversial to make assumptions about entailment unless they are very strongly supported. And in any case, if we do not want to call this a sense of 'possibility' then we can consider revising our definition of 'validity' in such a way that it refers to conceivability instead of possibility. This is more an issue of word use than anything. Whatever we call it, the idea is that some form of conceivability is at issue here.

Interpreting possibility in a wider sense than metaphysical or even conceptual possibility would allow us to account for the use of impossibilities in the evaluation of validity and it would allow inferences from "x is Peter's mothers brothers son. Therefore x is Peter's cousin" to fail as well thus discounting it as a logical truth.

There may be several advantages to such an account. First there the simple fact that argument per impossible has an incredibly long and rich tradition right up to the modern day with the endorsement, by many, of impossible worlds.[30] Endorsing this wider form of possibility allows for a charitable interpretation of this tradition rather than a mere dismissal.

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[30] For the history see Martin, Christopher J. (1992) and (1999). Several contemporary papers where the author makes use of or argues for impossible worlds are Kment, Boris (2006); Berto, Francesco (2007),

There is also the fact that certain so-called counterpossible statements have appeared to some to be true while other counterpossibles appear to be false. A counterpossible is a conditional statement whose antecedent is impossible. If we do not quantify over impossibilities then any counterpossible will come out trivially true, given that it is, by assumption impossible for it's antecedent to be true while it's consequent is false.

For instance, take the two statements "If someone proves Goldbach's conjecture people will be amazed" and "If someone disproves Goldbach's conjecture people will be amazed". Whether Goldbach's conjecture is true or false it is necessarily so. Suppose it is in fact false. Then the statement "If someone proves Goldbach's conjecture people will be amazed" has an impossible antecedent and would thus be trivially true on the standard interpretation. But the statement, "If someone proves Goldbach's conjecture the universe will explode" is also trivial true and yet it seems plausibly false. If we allow impossible worlds then we can specify worlds where Goldbach's conjecture is proved despite its necessary falisty. We could then compare the two counterpossible sentences at these impossible worlds. We might, for instance, compare the sentences by trying to determine which one is true at the closest impossible worlds. That is, if it's true that at the closest impossible worlds where Goldbach's conjecture is proved that people are amazed then we might declare this sentence true. If on the other hand it's not true at the closest impossible worlds where Goldbach's conjecture is proved that the universe then explodes we might declare this sentence false.[31]

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[31] Actual ranking of closeness of impossible worlds may prove impossibly difficult.

Plausibly, however, here too we would need an account of logical form since it will still be most plausible to think of logical validity as truth in virtue of form. And as pointed out above a satisfactory account of form still awaits us. In particular we need a satisfactory account of the logical constants, which has yet to be agreed upon.

One other problem with this idea is that there are many, many views different theoretical specifications of impossible worlds and many views as to which kinds should be considered. For instance, some impossible worlds, often called the non-normal worlds, merely differ from possible worlds in that modal truths like □A, which are, naturally, true at all possible worlds are false at the non-normal worlds. On other theories impossible worlds allow atomic formulas to vary in their truth-values as well. On such a view there could be worlds in which explicit contradictions like A ∧ ¬A hold. Still others assign truth-values to all formulas entirely independently of each other. In such worlds it could be that both A and B are true whereas A ∧ B is false.[32] It would require a lot of thought to decide which kind of worlds should be allowed into the picture. And until we have some idea about this, such an account will remain unsettled.

XI. Two Objections

There are doubtless many objections that could be raised against the arguments I have put forth in this paper. Here, however, I wish to address two specific objections among them. In the section covering *recapture* I mentioned that someone may wish to challenge the relevance of *recapture*.

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[32] See Berto, Francesco (2009) § 2

The first objection will be a specific challenge of this kind, arguing that *recapture* is irrelevant because although it does allow us to preserve the partition of valid and invalid arguments between two logics it does so by restricting the formulas that may be used and therefore only reproduces, for instance, classical validity for a restricted language. But the pluralist recognizes the classical validity for his entire language. As a result the pluralist may feel that even if the same sets of arguments are valid the circumstances under which this is true are different and thus *recapture* is entirely orthogonal to the issue of pluralism. This is because the pluralist recognizes a difference between language relative changes in consequence relation and multiple consequence relations within a single unrestricted language. As will be explained below, only the latter is a form of pluralism in Beall and Restall's sense.

The second objection goes as follows. Suppose all your objections against Beall and Restall's view of metaphysical modality is correct. Still once you allow quantification into impossible worlds, as you did in the last section, Beall and Restall's cases are once again viable candidates for consideration. Their view is then at least one possible view to take on what kinds of impossible worlds should be considered.

The first objection in more detail would go as follows. Suppose we have a language L for a logic L1 and there are two other logics L2 and L3 which are accepted by the logical pluralist. Suppose further that we may restrict L so that only a subset of its wffs are available, producing a new language L* and thereby recapturing L2. Further, we

may produce a different restriction L** that recaptures L3. The pluralists however

objects that his claim was that more logics besides just L1 were valid for L.

She may continue by noting that it's long been known that changing the

properties of a language can affect the number and kind of valid inferences that can be

produced.[33] The simplest example is a difference between first-order and second-order

languages. As Beall and Restall themselves point out in the language of second-order

logic we may formalize the sentence "If two objects have the same properties, they are

identical" thus, $\forall x \forall y (\forall X(Xx \leftrightarrow Xy) \rightarrow x=y)$. They go on point out that this sentence is

"valid in classical second-order logic. [However,] [i]f we do not have the machinery of

second-order logic at our disposal, our original sentence is not taken to be valid."(Beall &

Restall 2006 p. 79) So for instance it would not be valid in classical first order logic.

However, they wish to emphasize that "For us, pluralism can arise *within* a

language as well as *between* languages."(ibid.) They take language relative pluralism to

be uncontroversial and as they put it this "is not our kind of pluralism."(Beall & Restall

2006 p. 78). So given that *recapture* appears to offer a merely language relative

difference in the consequence relation the pluralist may well feel he is justified in

ignoring it.

In response it should first be noted that there are two readings of what the pluralist

is claiming here. One reading is more plausible than the other. Remember that we have

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[33] As Beall & Restall point out Carnap recognized that a logic consequence is relative to the properties of a formal language in the early to mid-20th century; Cf. (Beall & Restall 2006 p. 78). See for instance Carnap (1959).

a language L in which the arguments of logic L1 are valid. We claim to recapture L2 and L3 in restrictions of L, namely L* and L** respectively. On this reading the pluralist is saying that either L2 or L3, or both were already valid in L. But we have noted that the pluralist has already endorsed the idea that changes in language can affect the validity of arguments and thus affect the logic. Taking this into account this first reading seems less plausible.

Whether just any language change will affect the logic of a system or only specific changes depends on how finely we differentiate arguments. If, we differentiate them at the level of schemas we may be able to lose quite a few predicate symbols or variables without affecting the valid schemas. If however we think of arguments as sequents then the loss of any one predicate symbol results in the loss of all sequents, which contained that predicate. Similarly if the validity of an argument depends an actual word meaning, as in the red-colored example, losing that specific word will result in the loss of that argument and possibly a change in the consequence relation. But in any case once the connection between language change and logic is made it becomes less plausible that L2 or L3 could be preserved at both L on the one hand, and L* and L** respectively, on the other.

On a second reading the pluralist is merely claiming that some other logic L4 holds relative to the language L. But remember that in the original dialectical situation the pluralist is claiming four specific types of consequence relation hold, one each for cases as possible worlds, as models, as situations, and as constructions. If we can

recapture these the claim that some new logic holds relative to the original language is introducing an entirely new argument dialectically. This, second reading may be more plausible, prima facie, than the first but the pluralist will need to provide further argument to back up her claim that this new logic holds. When doing so she will undoubtedly appeal to the truth of different arguments in different cases.

At this point the dialectic will turn on my second and perhaps more central objection against the pluralist view of modality. Recall that I contended that the pluralists account of modality either collapses into an unwanted relativism, it threatens to become fictionalism about logical consequence, or it threatens to be wildly un-parsimonious.

If this objection is sound then my contention is that *recapture* offers us an alternate explanation of our ability to formulate these logics and understand what it would be like if they were valid. The pluralist's explanation then is just one among many and as such she would need to offer additional evidence in favor of her explanation. And this is where the objection against the pluralist's view of modality can play the role of evidence against her proposed explanation of the circumstances under which these logics are valid.

In addition there is some reason to be suspicious of the pluralist's contention that she has offered a sense in which several logics are available within a single language. Again, recall that cases, for the pluralist, are specified by specifying truth conditions. But

truth-conditions are also understood to provide the meanings of the logical connectives like conjunction and disjunction. In other words, they are often understood to be defining features of language. So there are in fact two ways to view what's going on when the truth-values of the atomic formulas of a language are being changed, as when we move from one model to another, or one possible world to another.

To see what I mean consider Tarskian models in particular. In his book *The Concept of Logical Consequence* John Etchemendy argued that modern conceptions of model theory are ambiguous. In Tarski's original sense changing from one model to another was understood as changing the interpretation or meaning of the symbols of formalized sentences. So, for instance, 'F(a) |= G(a)' evaluated across models would receive various interpretations of the meanings of F and G and 'a' would be understood to refer to different individuals. But the truth of these interpretations would, according to Etchemendy, be evaluated by looking at the actual world. So, if the sentences were interpreted in one model as meaning, "A is red. Therefore A is a rose" they could both be true provided that A refers to an actual red rose. But, because in a different model they would receive the interpretation where 'a' did not refer something that was included in the extension of F or G this would not be a valid argument. Still each different meaning was evaluated at the actual world. Etchemendy called this 'interpretational semantics'.

But there is also a way of seeing models as representing ways the world might be. To do this we would take one interpretation of 'F(a) |= G(a)' and hold it fixed. Suppose we interpret it to mean "A is red. Therefore, A is a rose" as above. We then

vary the extension of the predicates F and G as above but this time we take these to represent ways the world might be. There might have been fewer red things but more roses. There might have been no red roses, or all roses might have been red. This is what Etchemendy called representational semantics and it is similar to modern possible worlds semantics. The problem is that modern model theory does not always distinguish between these two ways of interpreting what's going on when a new model is specified. So in practice there is some ambiguity.

Now, it seems fairly certain that Beal and Restall see models, and in fact all cases, representationally. Recall that for the pluralist's cases are "'things' in which claims may be true"(Beall & Restall 2006 p. 89). But I would argue that given the ambiguity noted regarding models all of Beall and Restall's cases may similarly be read either representationally or interpretationally.

Here again we might appeal to the objection against the view of modality they are committed to. And we can go on to argue that given the implausibility of the pluralists view of modality it's implausible to see cases as ways the world might. Consequently the interpretational view of cases is more plausible. What we would be doing when considering a model in which not all red things are colored then is envisioning a situation in which the meanings of 'red' and 'colored' were such that they did not always overlap in their extension. If this is a plausible argument it allows us to counter the objection that *recapture* is irrelevant because it is merely linguistic whereas Beall and Restall's pluralism is not merely linguistic.

On the other hand, as we saw, the above responses depended crucially on the contention that Beall and Restall's view of modality is implausible. In the first case *recapture* merely left my account and the pluralists as alternate explanations of one phenomena. The objection against her view of modality allows me to argue that my explanation is superior. In the second case the objection against the pluralists view of modality allows me to argue that the interpretational view of cases is more plausible.

But the second objection to my own arguments is most problematic for my objection to the pluralists view of modality. In the final section of the paper I began to consider allowing a sense of logical modality, which recognizes impossible worlds. But, the objection goes, if we begin to allow impossible worlds are not the various types of cases Beall and Restall argue for likely to be admissible again.

In response I would only say two things. First, as I noted there are many, many ways of specifying impossible worlds. And it remains to be seen which ones will be necessary or useful in helping us to model intuitively correct arguments or sentences, e.g. the counterpossible sentences I mentioned. It's plausibly that these kinds of criteria will determine which impossible worlds we use, and the pluralist will have to argue for his cases along these lines. But even if he can argue that his various kinds of consequence are useful he will still have to overcome the *recapture* arguments above in the new impossible worlds context. And finally I would note that I argued that a kind supplementary constraint on logical consequence was necessary which would specify that

logical consequence is consequence in virtue of form. If this constraint is credible then Beall and Restall's account of consequence would still be ruled out.

I think what these objections and responses ultimately show is that much more needs to be said about the various types of modality and how each should be understood before we can rule out with any certainty the idea that there may be more than one kind of possibility and necessity appropriate for defining logical consequence. For example, would a definition of 'valid' in terms of the metaphysical impossibility be just as appropriate to our intuitive sense of logic as a definition in terms of epistemic impossibility. As I mentioned I think this would require examining the purposes and functions of logic in all its senses. In order to argue that only one definition of 'validity' is truly logical validity we would need to show that only that kind could serve the purposes intrinsic to logic. But fortunately I will have to put that task off till another occasion.

XII. Conclusion

I started out this paper trying to get a handle on what exactly the logical pluralist is committed to. I first tried to narrow down several senses of 'logic', separating a sense in which this refers to historically developed formal systems from a sense in which it appears to refer to an objectively holding relation, which we, in our theorizing, are trying to capture.

I then noted that the logical pluralist can be understood to be denying that there is strictly one answer to the question of which logic is correct. I tried to distinguish various senses of 'correct', e.g. moral correctness, epistemic correctness, according to which the question was being understood. I made some distinctions in the types of criteria we might use to choose a logic. I then pointed out that it may be correct to choose a logic in one sense but correct to draw a conclusion from certain premises within that logic in a different sense. I then argued that the logical norms are those that reflect the facts. Specifically so as not to beg any questions against the pluralist I suggested that we think of logic as those norms conforming to the facts regarding cases, whatever these turn out to be. Thus, plausibly, the criteria according to which we should choose a formal logic, a historically developed system, is the degree of that logic's reflection of the 'case facts'.

I then noted that there is a phenomenon called *logical recapture* according to which for some particular restriction in one's way of formulating premises one logic may produce all and only the valid arguments of another logic. I argued that this may present a problem for the logical pluralist in that he may need to show that there is no logic which can recapture all the various 'logics' he claims to have found or he will need to show why his account of their validity is superior to that of the theorist who embraces the recapturing logic. But I ultimately left this issue undecided given that *recapture,* on its own, leaves the pluralist and the logician who embraces the recapture of the pluralist's logics on equal footing. Which explanation of the validity of the logics in question is more plausible will depend on other factors.

Next I asked what exactly a case is, and thus ultimately whether the 'case facts' that a logic may need to conform to are really facts at all. I noted that Beall and Restall's pluralism with regard to modality either ends up committing them to relativism, or fictionalist view of modality and logical consequence, or an implausible number of different kinds of modality.

I considered a modified form of pluralism by taking the impossibility definition of validity and considering various interpretations of possibility, whether metaphysical, conceptual, or something else entirely. I also argued that even when we are quantifying over impossible worlds there is plausibly a sense in which we could say that these are 'possible' or at least 'conceivable'. Whatever the notion involved here there are some reasons to consider quantification over impossible worlds a plausible option. But I noted that each of these forms of validity would plausibly require a restriction so that only those arguments, which are valid in virtue of form count as truly logical. However, I noted that until we have an independently motivated account of the logical constants the prospects for a principled account of logical form will be dim.

I then addressed two objections. One objection argued that *recapture* is merely a linguistic phenomena and thus not problematic for the pluralist since she recognizes language relative differences in logical consequence as well. I argued that the very recognition of these differences makes the pluralists case more difficult than she perhaps realizes. And I argued that the pluralist herself can be read as having developed merely language relative differences in logical consequence. The second objection contended

that the pluralist's cases might become available again once impossible worlds are quantified over. I responded to this by noting that the issue will depend on which kinds of impossible worlds we find most useful in modeling intuitively correct arguments and is thus open. But I also noted that I made the case that logical consequence needs to be constrained with an account of validity in virtue of form thus ruling out Beall and Restall's cases anyway.

Ultimately, however we must leave the question of whether there is more than one true logic open, given the different interpretations of possibility. And even within each of these different senses the notion will not be well defined until we have an account of form.

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