4-24-2013

The Explanatory Indispensability of Mathematics: Why Structure is 'What There Is'

Nils Richards
University of Missouri-St. Louis

Follow this and additional works at: http://irl.umsl.edu/thesis

The Explanatory Indispensability of Mathematics: Why Structure is 'What There Is'

Nils Richards
M.A., Philosophy, University of Missouri – St. Louis, 2013
B.A., Philosophy, Classical Languages, University of St. Thomas – St. Paul, MN, 2011

A Thesis submitted to The Graduate School at the University of Missouri – St. Louis
in partial fulfillment of the requirements for the degree
Master of Arts in Philosophy

May 2013

Advisory Committee

Gualtiero Piccinini
Chair

Kent Staley

Judith Crane

Abstract

Inference to the best explanation (IBE) is the principle of inference according to which, when faced with a set of competing hypotheses, where each hypothesis is empirically adequate for explaining the phenomena, we should infer the truth of the hypothesis that best explains the phenomena. When our theories correctly display this principle, we call them our 'best'. In this paper, I examine the explanatory role of mathematics in our best scientific theories. In particular, I will elucidate the enormous utility of mathematical structures. I argue from a reformed indispensability argument that mathematical structures are explanatorily indispensable to our best scientific theories. Therefore, IBE scientific realism entails mathematical realism. I develop a naturalistic, neo-Quinean ontology, which grounds physical and mathematical entities in structures. Mathematical structures are the truth-makers for the entities of our quantificational discourse. I also develop an 'ontic conception' of explanation, according to which explanations exist in the world, whether or not we discover and model them. I apply the ontic account to mathematical structures, arguing that these structures are the explanations for particles, forces, and even the conservation laws of physics. As such, mathematical structures provide the fundamental grounding for ontological commitment. I conclude by reviewing the evidence from modern physics for the existence of mathematical structures.

To Willard van Orman Quine,

Who first warned me that Plato's beard has become gnarled, frequently dulling the edge of Ockham's razor.

Mathematics provides enormous utility to the sciences, ranging in application from equations used in the physical sciences, such as quantum mechanics, to the use of statistics in the life sciences, such as biology. The question is 'what follows about the ontology of mathematics from its application to science?' I aim to show that a substantive thesis concerning the reality of mathematical structures does follow, provided one adopts a realist attitude toward science. As a scientific realist, I take it that our best scientific theories are based on the principle of 'inference to the best explanation' (IBE). Our 'best scientific theories' are 'best' in virtue of IBE, so their 'success' is gauged relative to the principle as a baseline for scientific theories.

My claim in this paper is that scientific realism entails mathematical realism. By 'mathematical realism', I mean that mathematical *structures* are among 'what there is' in the universe. 'Structures' should not be confused with 'entities'. The latter are simply those things that "figure as values of the variables in our overall system of the world" in the sense of Quine: numbers, functions, sets, etc., as well as so called 'physical objects': hypothetical particles, and so on.[1] These 'values of variables' need to be grounded in order not to be 'abstract'. On my account, structures do the grounding.

I employ a mathematical naturalism as my ontology of mathematics—not Quine's Platonist naturalism, but naturalism in terms of mathematical structures, which are not abstract objects, but are fully grounded in the concrete, space-time universe. These structures are the truth-makers for statements about mathematical objects, and as such, meaningful reference to these objects is made possible.[2] My naturalistic approach is motivated by the ontological

---

[1] I use 'entity' interchangeably with 'object' throughout this paper.

[2] I follow Ross Cameron (2010) in this respect, who asserts that "what there really is is what *grounds* the true sentences describing the world: that is, the real existents are the *truthmakers* for the true sentences of English. So the ontologist's concern should be: what must the quantificational structure of the world be like to *ground* the true English claims we make?" My answer is that the quantificational structure is mathematical.

implications of modern physics, which suggest an ontology of structures, rather than objects and their properties. Although it remains difficult to pin down exactly *what* the structures *are*, physicists and philosophers currently look to structures of quantum physics, such as gauge symmetry invariants, mathematically characterized by the Casimir operators of gauge groups. Using group structures (the Poincare group, etc.), we derive structural properties with zero-value, like spin and charge, which are then ascribed to particles, such as the Higgs boson. The Higgs is not defined as an individual entity, but as a member of a mathematical structure—the gauge group of which it is a part. According to the current taxonomy of particle physics, zero-value properties "aren't merely absences of quantities or holes in being," but are "considered to be as real as non-zero value properties."[3] This, in outline, is my ontological approach.

My argument is defended according to an *explanatory* indispensability argument of the following form:

1. We ought to have ontological commitment to all structures that are explanatorily indispensable to our best scientific theories.

2. Mathematical structures are explanatorily indispensable to our best scientific theories. Therefore,

3. We ought to have ontological commitment to mathematical structures.[4]

A key aim in formulating the indispensability argument this way is to illustrate how the *existence* of something can be demonstrated by showing how it *explains* a phenomenon (e.g., how the

---

[3] Holger Lyre, "Structural Invariants, Structural Kinds, Structural Laws," forthcoming in *Probabilities, Laws, and Structures*, 3.

[4] This version of the argument is Mark Colyvan's (2001). As an entity realist, Colyvan follows Quine (1960, 1981). I have thus revised Colyvan's entity formulation to reflect the structural argument; I sharpen the notion of 'indispensability' via 'explanatory indispensability'; and I also weaken the 'all and only' clause in the first premise to 'all', leaving open possibilities besides structures for ontological commitment regarding scientific theories. The argument is otherwise the same.

existence of light quanta explains the photoelectric effect, how the molecular hypothesis explains Brownian motion, etc.); mathematical structures will be examined in this light.

Structural Grounding

Premise 2 states that mathematical structures are 'explanatorily indispensable' to our best scientific theories. 'Explanatorily indispensable' means that without these mathematical structures, our best scientific theories wouldn't be our 'best', since they would lack the requisite explanatory power. In this way, our best scientific theories *entail* mathematical structures. These structures *ground* the entities of our theories, according to a set of physical dependency relations. 'Grounding' is thus a form of *metaphysical* necessitation—a relationship which indicates that some levels of being are more fundamental than others, and as such, necessitate the less fundamental. Hydrogen and oxygen atoms are more fundamental than H_2O, insofar as H_2O *couldn't exist without* them. The grounding hierarchy is itself metaphysical, but direct dependency relations among phenomena are physical facts of nature. Like causation in the sciences, grounding is a system that backs explanatory patterns among facts in the world.

Consider the holes in Swiss cheese.[5] The holes are directly dependent on the shape of the cheese, in the sense that if there is no shape, there are no holes. And there is no shape if there are no deeper, physically constitutive facts, which ground the shape in more fundamental relations. Thus, the direct dependency of holes on shape *necessitates* levels of being. This is because the holes form a pattern, which directly depends on a more fundamental level of being—the level of shape. Grounding features thus serve as inputs to *structural* explanation. They tell us *why*

---

something has properties, according to further, more fundamental facts (structural) responsible for those properties.

In the sciences, grounding relations tell us what grounds what. In geology, microphysical features ground volcanoes, according to features like elevation, slope and stratification. In mathematics, structures ground objects such as numbers, according to features like ordinality, cardinality, oddness and evenness. The natural-number system is a mathematical structure that grounds the natural numbers. A structural relation of this system is the successor function: a relation between non-fundamental objects—the natural numbers—and the fundamental structure that grounds them—the natural-number system. Numbers are thus 'placeholders' in a mathematical structure. They are not self-subsistent things, but are directly dependent on structure. The structure provides the grounding relations for numbers. I claim we ought to have ontological commitment to mathematical structure for three reasons. The first is metaphysical: because the structure grounds the number; the second is logical: because scientific realism entails mathematical structures, hence mathematical realism; and the third is quantum physical, which I discuss in the following section.

<u>Mathematical Structuralism and Ontic Structural Realism</u>

Mathematical structuralism should be distinguished from ontic structural realism (OSR) in the following sense. OSR was originally formulated to answer underdetermination problems posed in quantum physics—problems like whether particles are individuals or non-individuals, whether quantum fields are substances or properties of space-time points, etc.[6] OSR is therefore not wedded to any particular doctrine concerning mathematical structures. However, the

---

[6] See French (2010) for an overview.

'eliminativist' version of OSR gives what I take to be the most conceptually coherent answer to the metaphysical underdetermination of physics: there are no objects at all, metaphysically speaking; structure 'is all there is'.[7] It's the most coherent because it seems best equipped to deal with the original underdetermination problems OSR was enacted to solve.

I believe OSR and mathematical structural realism are nonetheless engaged in a closely allied enterprise. In mathematics, I claim mathematical structure is 'all there is', in the sense that mathematical objects do not 'exist' independently of the structure that grounds them. In OSR, particles, like mathematical objects, do not 'exist' independently of the quantum physical structures that ground *them*. Over half a century ago, Quine pointed out that particles and mathematical objects are on equal ontological footing, but in any case, we need structures to ground entities. I believe physics and mathematics converge here, on the point of ontology. Like my account of explanatory indispensability, OSR looks to the best theories of modern physics to guide ontological commitment. Current physics suggests an ontology of structures over objects. Thus, I share with OSR the commitment to show that there are only structures.[8]

<h2 style="text-align:center">The Indispensability of Mathematics to Science</h2>

Following Frege's 1898 observation that "it is applicability alone which elevates arithmetic from a game to the rank of a science,"[9] the thought that indispensability puts mathematics on par with science began to take hold. This motivated a realist conception of the *utility* of a 'successful' theory that generated novel prediction, tractability, economy, etc. By 1947, Gödel was characterizing this 'success' in terms of set theory, noting that by 'success' he

---

[7] See Ladyman and Ross (2007), p.130, for the original formulation.
[8] If my ontology of mathematical structures is correct, there is ultimately nothing *distinctively mathematical* about them, since they are simply the quantum physical structures of physics; however, since they are fully described mathematically, there is nothing *distinctively physical* about them, either.
[9] Gottlob Frege, *Philosophical Writings* (Oxford: Basil, 1960), 187.

meant "fruitfulness in consequences, in particular in 'verifiable' consequences," concluding that the axioms of set theory "would have to be accepted at least in the same sense as any well-established physical theory."[10] Gödel's point is that in virtue of set theory's utility value, we must treat the entities of set theory *as entities of a physical theory*. The entities of set theory (axioms, definitions, etc.) are thus quantified over in any scientific theory to which they are applied, and are accordingly 'verified' along with mass, electron, etc. This empirical verification of *applied* mathematical entities leads to Putnam's 'hunch' that mathematics is closer to an empirical science than an a priori discipline.[11]

The modern incarnation of the indispensability argument is due to Quine, who held that in our ontology, we "need to add *abstract* objects, if we are to accommodate science as currently constituted," since "things we want to say in science may compel us to admit into the range of values of the variables of quantification not only physical objects but also classes and relations of them."[12] This is so, Quine thinks, because mathematics "is best looked upon as an integral part of science, on a par with the physics, economics, etc., in which mathematics is said to receive its applications."[13] Insofar as Quine means that scientific realism entails mathematical realism, I am in agreement; but as I argue in this paper, we need only accommodate the structures that ground Quine's *abstract* objects—numbers, sets, classes, etc. This strategy circumvents Quine's 'hyper-Pythagoreanism'[14] by grounding objects in structures.

---

[10] Kurt Gödel, "What is Cantor's Continuum Problem?" in *Philosophy of Mathematics*, eds. Paul Benacerraf and Hilary Putnam.

[11] See Putnam (1975).

[12] W.V. Quine, *The Ways of Paradox*, 231.

[13] Ibid, 231.

[14] At this time (1976), Quine was holding to an ontology of pure set theory, the triumph of which "has to do with the values of the variables of quantification, and not with what we say about them…The things that a theory deems there to be are the values of the theory's variables, and it is these that have been resolving themselves into

Psillos-Style Scientific Realism vs. Laudan's Pessimistic Induction

I follow Psillos in taking our best science as the most reliable guide to our ontological commitments.[15] Given a 'successful' scientific theory, IBE needs to yield novel successful predictions of phenomena in a sound, reliable fashion. Larry Laudan's attack on the 'success' of IBE, per his 'pessimistic induction', weakens the link between true theories and success, but leaves open the possibility that a scientific theory could still be approximately true. If *some* of a theory's assertions bear an explanatory connection between empirical success and the theory's being right about the unobservable world, we have reason not to take Laudan's pessimistic induction seriously.

The Ontic Conception of Explanation

The force of the IBE argument derives from the nature of explanation. In cases of genuine explanation, we have successfully delimited a portion of the objective structure of the world. This is the 'ontic conception' of explanation, which I defend. According to this conception, Newton's explanation of the tides is deep and powerful because he uses the tides themselves to explain his theory of universal gravitation. By appealing directly to tidal acceleration, etc., the explanatory power of gravitational attraction goes up, providing deeper evidence for its existence. Newton does not use inferences, arguments or models. This is a matter of the *direction* of explanation in giving an account of how explanation works. We move *from* the world *toward* a theory. In Newton's theory of the tides, it is the moon's gravitational attraction itself that explains them.

---

numbers and kindred objects—ultimately into pure sets. The ontology of our system of the world reduces thus to the ontology of set theory, but our system of the world does not reduce to set theory." (Quine [1976], p. 503)

[15] Psillos (1999).

Turning to neuroscience, let's consider Carl Craver's causal-mechanical explanation of neurotransmitter release.

> The mechanism begins…when an action potential depolarizes the axon terminal and so opens voltage-sensitive calcium (Ca^{2+}) channels in the neuronal membrane. Intracellular Ca^{2+} concentrations rise, causing more Ca^{2+} to bind to Ca^{2+} /Calmodulin dependent kinase. The latter phosphorylates synapsin, which frees the transmitter-containing vesicle from the cytoskeleton. At this point, Rab_3A and Rab_3C target the freed vesicle to release sites in the membrane. Then v-SNARES (such as VAMP), which are incorporated into the vesicle membrane, bind to t-SNARES (such as syntaxin and SNAP-$_{25}$), which are incorporated into the axon terminal membrane, thereby bringing the vesicle and the membrane next to one another. Finally, local influx of Ca^{2+} at the active zone in the terminal leads this SNARE complex, either acting alone or in concert with other proteins, to open a fusion pore that spans the membrane to the synaptic cleft.[16]

Craver appeals directly to the mechanism to explain the phenomenon. This is because the mechanism *is* the explanation of neurotransmitter release. What Craver has done is correctly read off the causal structure of the mechanism, such that we see how it works. It would be strange to think that the entities and activities involved in this process somehow depend on an observer to explain the mechanism; for instance, that t-SNARES are not incorporated into the axon terminal membrane unless there is an observer present.

Furthermore, like Newton's explanation, there are no inferences, arguments or models involved. That's because none of these qualify as genuine explanations. An inference like 'the salt dissolved because it was placed in water' is merely an inference with little explanatory value. An argument like 'all gases expand when heated; x is a gas; so, x will expand when heated' merely states a law-like regularity with a particular instance subsumed under it, and concludes with a future prediction. A model like Hodgkin and Huxley's action potential, while historically important for neuroscience, is not genuinely explanatory. Hodgkin and Huxley are themselves clear on this point, noting that the agreement between their model and the voltage-clamp data

> must not be taken as evidence that our equations are anything more than an *empirical description* (my italics) of the time-course of the changes in permeability to sodium and potassium. An equally satisfactory description of the voltage-clamp data could no doubt have been achieved with equations of very different

---

[16] Carl Craver, *Explaining the Brain* (Oxford: Oxford UP, 2007), 5.

form, which would probably have been equally successful *in predicting* (my italics) the electrical behavior of the membrane.[17]

Explanations must do more than describe and predict phenomena, in order to count as explanations. They don't merely *answer* a why-question; they actually tell us *why*.[18] In this way, explanations yield objective features, while phenomenological models yield only empirical adequacy. If we want to get the phenomena *right*, and not merely *save them*,[19] we have to understand that mechanisms and mathematical structures are themselves the explanations, and that models of these explanations are pragmatic approximations, at best. Hodgkin and Huxley save the phenomena of the action potential with mathematical modeling, but leave the explanations of the underlying mechanisms for future generations.

The world is composed of explanatory information. For Wes Salmon, this information is Mark transmission: self-propagating causal processes with certain structural frequencies that persist beyond the point of intervention. This account follows the ontic conception, initially developed by Alberto Coffa, and later defended by Salmon.[20] The central idea is that in stating 'the moon's gravitational attraction explains the tides', we assert that gravitational attraction is "out there in the physical world," understanding that these objective features "are neither linguistic entities (sentences) nor abstract entities (propositions)."[21]

Although the causal-mechanical approach nicely elicits the ontic conception of explanation, explanation need not be causal-mechanical. While this kind of explanation can fruitfully be applied to biology and chemistry, matters are quite different in physics. That's

---

[17] A.L. Hodgkin and A.F. Huxley, "A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve," *Journal of Physiology* 117 (1952), 541.
[18] See Salmon (1978) for the fully developed account.
[19] See Van Fraassen (1980).
[20] See Coffa's dissertation (1973) for the original proposal of the ontic conception of explanation.
[21] Wesley Salmon, *Four Decades of Scientific Explanation* (Minneapolis: U of Minnesota Press, 1989), 86.

because unlike 'bottom-up' causal-mechanical explanations, which explain general regularities like Boyle's law with the kinetic-molecular theory of gases, explanations in physics are often 'top-down'. These explanations appeal to something very general, like energy conservation, to explain a range of interactions among various phenomena, thus *unifying* them, rather than explaining them in terms of *causes*. Some thirty-five years ago, Mark Steiner pointed out that

> we do not learn about the neutrino by transmission of energy from the neutrino to us—the neutrino is very difficult to detect by direct interaction. Indeed, as far as is known, beta decay is noncausal—no anterior event causes the breakup of the unstable lithium 6 nucleus. Nor does the neutrino participate in any event which causes the other particles' motion—through which we infer the existence of the neutrino. Beta decay 'just happens' in accordance with the law of conservation of momentum, enabling us to infer a new particle. *Laws of conservation are simply not causal laws. They provide constraints on what is allowed to happen* (my italics).[22]

The idea is that the *constraints* of conservation laws are *not causes*, but are nevertheless genuinely explanatory in many cases. This is because of their power to constrain fundamental forces. When electric and gravitational forces both conserve energy, it can't be that these force laws explain why energy is conserved; the common explanation must be found in the law of energy conservation itself. On my account, mathematical structures are very closely akin to conservation laws, in the following sense. Conservation laws are entailed by invariance principles, and these principles play a central role in space-time symmetries, such as global gauge symmetries ('global' = depends on constant parameters). In fact, Emmy Noether's first theorem[23] states that 'for every continuous global symmetry there exists a conservation law'. If these invariance principles turn out to be mathematical structures, it follows that conservation laws are entailed by mathematical structures, insofar as such laws are explained in terms of structures.

---

[22] Mark Steiner, "Mathematics, Explanation, and Scientific Knowledge," *Nous* 12 (1978), 22.
[23] See Tavel (1971) for the English translation of this important 1918 work.

Graham Nerlich's 'geometric style of explanation' uses the curvature and variable curvature of space-time as explanatory *constraints*, rather than *causes*.[24] On his account, material particles causally interact, but their interactions are limited to select paths, directions and distances brought about by the geometric constraints of space-time. Thus, the mathematical structure of space-time is capable of explaining the behavior of matter without causing it. He notes that General Relativity

> provides a very strong example of geometric explanation since not only is spacetime curvature the fundamental explanatory concept of the theory, but the idea of spacetime geometry is actually used to reduce causal explanation by gravitational force in space during time. If spacetime is flat (i.e. Minkowskian or pseudo-Euclidean) then a geodesic or linear path in spacetime projects onto a motion, uniform in time, along a geodesic or linear path in space.

Nerlich holds that in many cases, the geometry of space-time does the explaining, not causes within it. Acceptance of his argument depends in part on one's view of *what* space-time *is*, and what it's capable of on its own.

The point has been to respond to the causal-mechanical approach to explanation, which works well in some sciences, but not in physics. I agree with Salmon's estimate of top-down and bottom-up explanations: each offers explanatory virtues. The ontic conception of explanation applies to both, bringing us back to the IBE argument for scientific realism. The whole point of 'inference-to-the-best-*explanation*' is that in making such inferences, we are approximating the way things stand in the world *through explanation*. By appealing directly to a mechanism or mathematical structure, rather than an argument or model, we recognize that explanations are part of the structure of the world. When we correctly read off the explanation, we call it our 'best'.

---

[24] Graham Nerlich, "What Can Geometry Explain?" *British Journal for the Philosophy of Science*, 30 (1979), 80.

Truth, Objectivity and Explanation

This accords with Whewell's 'coherence theory of truth', which posits that as a successful theory develops, it tends to explain an increasing range of empirical phenomena, due to a natural core of principles in the theory. The principles will depend on the kind of science we're working with. It seems to me that truth cannot be divorced from explanation in any non-trivial sense. This is because on my view, explanation does not equate to empirical adequacy, since empirical adequacy yields only description and prediction. So, if truth cannot be divorced from explanation, and a theory's success is gauged relative to its explanatory power, then a theory will be successful to the extent that it approximates truth. We gauge this extent by the depth and range of phenomena explained by the theory.

I follow Armstrong in his conjecture that the explanatory power of mathematics is the key justifying principle for why mathematics is indispensable to our 'best theories', and hence to the IBE form of scientific realism.[25] This leaves open the possibility that our 'best theories' entail mathematics as a means of *describing* the objective world, only. The relevant mathematics is not itself *part of the objective world*. In this way, Armstrong's account is consistent with mathematical fictionalism, since the fictionalist can be a realist with respect to our 'best theories', while denying reality to mathematics. Contrary to this position, I argue mathematical structures are *part of the objective world itself*, and hence *explain* that world; therefore, our 'best scientific theories' entail mathematical realism. Our best science, not metaphysics, is the most reliable guide to ontological commitment. On this account, we use explanatory power to establish existence, and thus to sanction ontological commitment.

---

[25] Armstrong (1989) pp. 7-10.

Ontological Commitment to 'What there Is': A Case Example of Mathematical Structure

Assuming one of our best scientific theories is Einstein's Theory of Special Relativity, the question is 'what ontological commitments does the theory involve'? We know the Lorentz Transformations form the kinematic basis of Special Relativity. They are linear relations between events, where we can think of an event as simply a point in space-time. The fundamental assumption that will determine the Transformations is that space-time is homogenous. We must also assume there are symmetries of space-time and that rotations in space-time should be invariant.

In the most elementary sense, the assumptions boil down to an underlying notion of *symmetry*: essentially a 'form-invariance' principle, which allows for the transformation of variables that leaves the explicit form of the equations unchanged. Symmetry is, in this sense, a structural preservation mechanism for the automorphism structures in which particles are grounded. This mechanism allows us to characterize the structure of space-time, according to Lie algebra, Riemannian geometry, etc. As a concrete space-time mechanism, symmetry yields invariance under transformation, thus providing a powerful explanatory account of the physical basis of automorphic structures—prime candidates for mathematical structures that ground space-time particles.

Einstein's theory thus involves an ontological commitment to *symmetry*—the space-time mechanism which yields and preserves *structure*. If you're a realist about events (space-time points), then according to the theory's ontological commitment, you're a realist about the *symmetry* in which those events are embedded, hence the *structure* that grounds the events. Recall that the theory states that transformations of relations between these events necessarily

occur within the space-time symmetry. Thus, insofar as symmetries of Special Relativity are mechanisms for preserving physical structures, the scientific theory entails ontological commitment those structures, and hence Special Relativity entails mathematical realism.

Objection from the 'Special Sciences'

One objection to the indispensability of mathematical structures is that examples from physics are more transparent than other branches of science, such as the life sciences. If our 'best scientific theories' come from all areas of science, not just physics, it isn't clear how mathematics plays a comparable explanatory role in these areas. While I agree that physics examples are more easily produced, I disagree that mathematics can't play a comparable explanatory role in the life sciences. At least *some* theories of the life sciences achieve 'success' only by appeal to the explanatory power of the mathematics involved.

Take Alan Baker's case study on North American cicadas.[26] One of the goals of the study was to show why the life-cycle of the 'periodical' cicada is prime-numbered (13 and 17 years). Of the two evolutionary explanations offered by the biologists (one involving avoidance of predators, the other based on avoidance of hybridization with subspecies), both appeal explicitly to number-theoretic results, involving the notion of a 'lowest common multiple', as well as the intersection of prime number periods. Baker claims the explanations ineliminably involve reference to a *coprime* theorem that is genuinely explanatory, when considered as essential to the overall explanation given by the biologists. If we remove the theorem, the explanatory power of the theory is significantly weakened.

---

[26] Alan Baker, "Are there Genuine Mathematical Explanations of Physical Phenomena?" *Mind* 114 (2005).

From this case of genuine mathematical explanation, Baker draws the conclusion that 'numbers exist'. Baker's conclusion is not without its criticisms.[27] The details of the case, concerning whether equally strong non-mathematical explanations could be given, are currently under debate. Furthermore, I argue that Baker's numbers must be grounded in structures. But the example still suggests that mathematics *does* play a genuinely explanatory role in the life sciences, not just in physics. The cicada case thus provides reason to take mathematical realism seriously, according to its explanatory application in the life sciences.

<u>The Right Kind of 'Abstraction': Mathematical Idealization and Explanatory Depth</u>

Consider Batterman's 'asymptotic explanation' of the rainbow.[28] He starts by asking how it is that the circumstantial details (wind, raindrop shape, etc.) of rainbows are always unique, yet it is always the same structural pattern we witness. To explain this, we appeal to patterns and regularities. We have to know why rainbows always appear with the same light intensities and spacing between bows (among other things). Batterman uses the 'asymptotic' approach, which reveals how mathematical relations provide *limiting cases* for equations drawn from two different theories: classical and quantum mechanics. This means that certain mathematical structures shared by the two theories (bridged by semiclassical mechanics) remain stable under perturbations, and account for the light intensities and bow spacings in the same way.

The stability of the mathematical structures is mirrored in the rainbow, which can undergo perturbation of shape by raindrops and other factors, according to a wave equation in the limit, whereby the wavelength of light approaches zero. Robust and repeatable patterns emerge

---

[27] See Saatsi (2007, 2011).
[28] Robert Batterman, *The Devil in the Details* (Oxford: Oxford UP, 2002).

from applied mathematics, deepening our understanding of the nature of such regularities and invariances. For Batterman, the explanatory power of the theory of geometrical optics (ray theory) and wave theory combined allows us to discard many of the physical details of the rainbow in favor of mathematical idealization. Using these theories together with asymptotic reasoning, Batterman explains where the arc of a rainbow comes from, predicts where the arc will be located, and accounts for the intensities and locations of various bows we see. He argues that these features of rainbows can *only* be explained by a combination of mathematics (the 'limiting operation', geometry, etc.) and physical theory.

Batterman's asymptotic reasoning distinguishes useful idealization of the rainbow from its literally realistic features. His 'asymptotic explanation' of supernumerary bows combines G.B. Airy's integral equation with the Stokes Phenomenon. The explanation shows that the details supposedly distinguishing one physical system from another are actually *irrelevant* when we 'abstract away' from the systems themselves, in favor of understanding their universal behavior in terms of the fundamental physics which grounds all physical systems. As Batterman argues, the fundamental physics ineliminably involves accounting for the physical phenomenon (a rainbow) at the right level of *mathematical* abstraction. These are cases in which the explanatory indispensability of mathematics entails ontological commitment, given the role played by the mathematics in the theories.

Like Batterman, Weslake's 'abstractive account of explanatory depth' takes abstraction to be a genuine dimension of explanatory power, yielding some of our 'best explanations' of scientific theories.[29] 'Explanatory depth' reveals the objective features of explanations by showing how 'depth' in no way depends on models or representations. This approach to

---

[29] Brad Weslake, "Explanatory Depth," *Philosophy of Science* 77 (2010).

explanation tracks the attitudes of scientists. As Weslake notes, "The abstractive account provides an objective notion of explanatory virtue that justifies this claim, without requiring recourse to subjective notions of simplicity and understandability."[30]

Mathematics is explanatorily indispensable to IBE because of its ability to unify through generality. We often obtain deeper explanations by 'abstracting away' from the messy details. In doing so, we achieve greater explanatory range. Unlike D-N (deductive-nomological) models of scientific explanation, in which the mathematics is merely a component of the argument, the mathematics in Weslake's account generates "explanations that would remain explanatory even if the fundamental laws of nature were different, within a certain set of constraints, from what they actually are."[31] In this way, mathematical explanation applies to an impressive range of nomologically *and* logically possible situations.

'Pure' versus 'Applied' Mathematics: Can Empirical Science Entail Mathematical Truth?

Christopher Pincock holds that mathematics' contribution to scientific explanation is not justified empirically, but only on 'a priori' grounds. He asserts that "mathematical claims can only contribute to explanations if the mathematical claims are independently supported by purely mathematical means," concluding that mathematics is 'conditionally' indispensable to science, insofar as "its claims receive substantial support independently of its application in science."[32] From this it follows, for Pincock, that mathematical claims are a priori, while claims concerning unobservables are empirical. The problem of reconciling mathematical and scientific claims is that "the truth of many mathematical claims goes far beyond what is needed for the explanation

---

[30] Ibid, 289-290.
[31] Ibid, 291.
[32] Christopher Pincock, *Mathematics and Scientific Representation* (Oxford: Oxford UP, 2012), 211.

to be successful."[33] Pincock's point is that mathematical truth and empirical success are distinct concepts. Drawing ontological conclusions from their co-occurrence in a scientific explanation is a mistake.

I grant Pincock that the domain of pure mathematics is indeed separate from empirical science. But this point does not threaten my overall argument. Recall that the explanatory indispensability argument singles out *applied* mathematics as the driving force behind ontological commitment. As far as I'm concerned, mathematical structures posited by pure mathematics 'exist' only insofar as they receive application in the sciences. A Euclidean vector, for example, exists insofar as it describes the velocity and acceleration of a traveling particle— but it does not 'exist' *a priori*, as a purely mathematical posit. We cannot determine pure mathematical truths by empirical science because such truths have *intra-mathematical* explanations—explanations which in no way depend on the empirical world. Whether we can discover such truths empirically is an interesting question, but whether *we can know mathematical truths empirically* and whether *mathematical truths are empirical*—these are really two different questions (outside the scope of this paper).

In Putnam's essay 'What is Mathematical Truth?' he argues that mathematics is not justified by proof/induction (i.e., by purely formal means), but by 'success' in physical science. The contrast I wish to draw here is that Pincock and Putnam represent the Scylla and Charybdis of taking the whole of mathematics' relation to the world as a disjunction—either analytic or synthetic. Pincock tends to think of mathematics as analytic, while Putnam stresses its synthetic character. Twentieth-century mathematics shows that neither view is strictly correct.

---

[33] Ibid, 212.

Treating mathematics in its reliance on a priori justification, Pincock clarifies that in providing an explanation, "scientists first justify the relevant mathematics by mathematical means and then use this mathematics to explain scientific phenomena."[34] He raises two related points. First, scientific explanation cannot yield mathematical truth because this can only come from within mathematics. Second, because of this, we face an underdetermination problem in trying to use our best scientific theories to provide evidence for the truth of mathematical claims. Given these factors, IBE scientific realism cannot entail mathematical realism, because our best scientific theories do not entail mathematical truth.

<u>Explanatory Indispensability over 'Truth': the Case for Mathematical Trivialism</u>

It's not immediately clear that our best scientific theories *need* to provide such evidence for the truth of mathematical claims. Some portion of mathematics behind the Theory of Special Relativity might turn out not to be justified, either due to the theory itself, or to something internal to the mathematics—but this doesn't mean the theory isn't true or approximately true. Nor does it mean mathematics is no longer explanatorily indispensable to the theory (hence entailed by IBE). It might turn out, for example, that mathematical truths are 'trivially true', as Cameron and Rayo have recently suggested.[35]

On this account, mathematical truths make no demands of the world that they exist. So the truths themselves do not entail ontological commitment, and hence truth-makers are not required for mathematical truths. Take Goldbach's conjecture, 'every even number greater than 2 is the sum of two primes'. Under trivialism, no truth-makers are required because nothing about the world needs to be satisfied to make the conjecture come out 'true'; consequently, no

---

[34] Ibid, 216.
[35] See Ross Cameron (2008, 2010) and Augustin Rayo (2009) for detailed work in this area.

ontological commitment is accrued. In principle, Goldbach's conjecture (the structural aspect) could still be explanatorily indispensable to some of our best scientific theories. If so, we ought to be ontologically committed to its structure, not because of its formal truth, but because of its explanatory indispensability.

The Honeycomb Theorem

The question of how pure mathematics is justified by mathematicians is not my concern in this paper. Once justified, pure mathematics can be applied. Take an example from Lyon and Colyvan. Like Baker, they connect a mathematical theorem to a biological claim, yielding an explanation. They focus on the question, 'why does a hive-bee honeycomb have a hexagonal structure?' What needs to be explained is "why the honeycomb is always divided up into hexagons and not some other polygon (such as triangles or squares)."[36] Biologists make assumptions concerning evolutionary advantage, and then build into their explanation the hypothesis that the hexagonal grid wastes less wax than other shapes, so the bees that use the grid are selected over bees that waste energy building less efficient combs. Then they conjoin the biological claim to what's called the honeycomb theorem, proved by Thomas C. Hales in 1999: "A hexagonal grid represents the best way to divide a surface into regions of equal area with the least total perimeter."[37]

Hales's proof explains why the hexagonal grid provides an optimal means of dividing up a two-dimensional surface so that less wax is wasted and less energy spent. So, the evolutionary biology, coupled with the honeycomb theorem, gives us what Lyon and Colyvan refer to as our 'best explanation for this phenomenon'. We can only assume the geometry is justified, and that

---

[36] Aiden Lyon and Mark Colyvan, "The Explanatory Power of Phase Spaces," *Philosophia Mathematica* 16 (2008), 228.
[37] Ibid, 229.

intra-mathematical problems will be worked out intra-mathematically. We then conclude that the mathematics is explanatorily indispensable to the explanandum phenomenon, in this case, the structure of the hive-bee honeycomb.

<div align="center">Ontological Commitment and IBE: The Fundamental Thesis</div>

The explanatory power of mathematics brings us back to the notions of approximate truth, success, and the IBE argument for the entailment of mathematical realism. Our best scientific theories, which are true or at least approximately true, tell us that ontological commitment to some portion of mathematics is unavoidable; without ontological commitment to the truth of our best scientific theories, and hence without relying on these theories as a guide to ontological commitment, the commitment to mathematical realism would be unclear; but we *do* make the ontological commitment to the truth of our best scientific theories, insofar as we adopt the IBE scientific realist stance. In this way, scientific realism entails mathematical realism.

What does this 'portion of mathematics' amount to? As the explanatory indispensability argument states, 'mathematical structures'. Arthur Eddington held that for a 'strict expression' of structural knowledge, a mathematical form was essential. This is because "structural knowledge can be detached from knowledge of the entities forming the structure," enabling us to "introduce spherical space in physics," according to which "we refer to something—we know not what—which has this structure."[38] Eddington indicates the type of structural grounding elucidated in this paper. Structures are ontologically fundamental, so we can genuinely refer to something like spherical space as a *structure*, not an *object*, without committing ourselves to the existence of an object—spherical space.

---

[38] Sir Arthur Eddington, *The Philosophy of Physical Science* (Ann Arbor: U of Michigan, 1958), 143, 146.

Premise 2 of my original argument states that 'mathematical structures are explanatorily indispensable to our best scientific theories'. Adopting Shapiro's 'relationalist' notion of mathematical 'structure', we could take it to be "the abstract form of a system, highlighting the interrelationships among the objects, and ignoring any features of them that do not affect how they relate to other objects in the system."[39] The problem with this conception is that it isn't clear how 'relations without relata' makes sense. Furthermore, if by 'abstract form' Shapiro means mathematical structures are not fully grounded in the concrete space-time universe, his account falls short of my naturalistic requirement that mathematical structures must be the truth-makers for statements about mathematical objects, rather than 'abstract' in the Platonist sense.

Consider the natural number 5. The Quine-Putnam indispensability argument decrees that in 'quantifying over' the number 5 we are confirming its existence and thus ought to be ontologically committed to it;[40] however, on the structural account, the number 5 cannot be explanatorily indispensable to our best scientific theories unless it is considered part of the natural-number system, with a structural position in it. The structural component of the number 5 *can* be explanatorily indispensable, insofar as it is grounded in the natural-number structure.

Mathematical structuralism stems from a close attention to the current scientific climate. In quantum physics, 'group structure' plays an indispensable explanatory role. These types of structure are central to theories of gauge symmetry, invariance principles, objectivity of structure, etc. It is far from clear, however, that quantification over a set of numbers requires ontological commitment to each and every number of the set *plus* commitment to the *set itself*. Abstracta such as numbers and sets violate what Ladyman and Ross call the 'principle of

---

[39] Stewart Shapiro, *Philosophy of Mathematics* (Oxford: Oxford UP, 1997), 74.
[40] See Putnam (1971) p.57 for the canonical formulation.

naturalistic closure' (PNC):[41] metaphysical claims earn their keep by explanatory contribution to a set of scientific hypotheses (at least one of which is drawn from physics), such that the metaphysics strengthens the explanatory power of the hypotheses, when joined together. As I have shown in this paper, mathematical structuralism accords quite well with the PNC, though the principle is too strongly stated that one of the hypotheses *must* be drawn from physics.

Quantifying over 'the set of all electrons' does not strengthen the explanatory power of the theory of electrons in which it appears. On the other hand, quantifying over the structural properties of electrons *does* strengthen the theory's explanatory power, provided we can ground the properties in the appropriate physical structures. The PNC is a latter-day extension of Quine's own naturalistic principles, but it rules out his method of quantification, since quantifying over abstract objects fails to strengthen explanatory power. As Steven Weinberg points out, "mathematical structures that physicists develop in obedience to physical principles have an odd kind of portability. They can be carried over from one conceptual environment to another and serve many different purposes." He concludes that these structures are often found "to be extraordinarily valuable by the physicist."[42]

The idea is that mathematical structures have a life of their own, independent of the interests of the physicist who uses them. That mathematical structures can be 'carried over from one conceptual environment to another' suggests a genuine reference to them—that their existence persists through theory change.[43] The enormous flexibility of mathematical structures,

---

[41] *Everything Must*, 37.
[42] Steven Weinberg, *Dreams of a Final Theory* (New York: Pantheon, 1992), 152.
[43] Although preservation of structure through theory change is not explored in this paper. For the canonical account of 'epistemic structural realism', see Worrall (1989).

highlighted by Weinberg, brings us back to the utility value of mathematics and what ontological conclusions can be drawn.

Conclusion

I conclude by recalling Eugene Wigner's well-worn phrase concerning the 'unreasonable effectiveness of mathematics in the natural sciences': "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve."[44] As I have tried to show throughout this paper, the 'appropriateness' is a matter of physical fact, not a 'miracle', since mathematical structures are fully instantiated in the concrete space-time universe. They serve as truth-makers for mathematical statements, and these statements render our formulations of the laws of physics true, or at least approximately true. For these reasons, there is nothing 'unreasonable' about the 'effectiveness' of mathematics.

The insight that mathematical structures are fully 'physical' goes the distance in helping us understand Wigner's 'gift' of the language of mathematics. By 'reading off' this language from mathematical structures, we achieve great explanatory power in our scientific theories, particularly in quantum physics. If the principle of 'invariance under transformations' is sufficiently explained by mathematical structures, we conclude, on the basis of explanatory power, that those structures exist. Thus, they become explanatorily indispensable to our best scientific theories.

As for explanation, I have argued for the ontic conception: explanations are full-bodied features of the world. Mathematical structures are themselves such explanations. These

---

[44] Eugene Wigner, "The unreasonable effectiveness of mathematics in the natural sciences," *Communications of Pure and Applied Mathematics* 13 (1960): 14.

structures are fundamental. They ground mathematical objects and serve as truth-makers for statements about them. To recap, we ought to have ontological commitment to mathematical structures for three reasons: because they ground what we posit about the world; they are entailed by scientific realism; and because there is powerful evidence for them in quantum physics. The idea that mathematical structures are among 'what there is' has become commonplace in physics today. Max Tegmark has recently claimed that 'reality itself is a mathematical structure'.[45] He argues for a monism about mathematical structure, according to which

> instead of having one mathematical description for this and a different one for that, we realize there's a single mathematical structure that encompasses all of it…it would be a natural conclusion…if there's a single mathematical structure that *is* our reality, and all the mathematical structures that we've discovered before are part of this more beautiful whole.[46]

I adopt Batterman's approach to explanatory contact between mathematics and natural science, which "looks to the world as the 'driving influence' for how mathematics gets applied, rather than to fortuitous parallels or analogies between mathematical structures and physical structures."[47] Batterman concludes, against Wigner's 'miracle', that "the world itself tells us that a certain kind of mathematical language is required for genuine understanding."[48] By 'mathematical language', Batterman means mathematically powerful *idealizations* that yield rich, explanatory patterns among physical phenomena. Making such connections reveals that mathematics is explanatorily indispensable to our best scientific theories, and therefore, that IBE scientific realism entails mathematical realism (structural).

[45] "Do We Live Inside a Mathematical Equation?" *Science Now*, February 16, 2013.
[46] Ibid.
[47] *On the Explanatory Role*, 24.
[48] Ibid, 24.

Bibliography

Armstrong, David. *A Combinatorial Theory of Possibility*. Cambridge: Cambridge UP, 1989.

Baker, Alan. "Are there Genuine Mathematical Explanations of Physical Phenomena?" *Mind* 114 (2005): 223-238.

Baker, Alan. "Mathematical Explanation in Science." *British Journal for the Philosophy of Science* 60 (2009): 611-633.

Batterman, Robert W. *The Devil in the Details: Asymptotic Reasoning in Explanation, Reduction, and Emergence*. Oxford: Oxford UP, 2002.

Batterman, Robert W. "Into a Mist: Asymptotic Theories on a Caustic." *Studies in History and Philosophy of Modern Physics* 28 (1997): 395-413.

Batterman, Robert W. "On the Explanatory Role of Mathematics in Empirical Science." *British Journal for the Philosophy of Science* 61 (2010): 1-25.

Cameron, Ross. "Quantification, Naturalness, and Ontology." In *New Waves in Metaphysics*, edited by Allan Hazlett, 8-26. London: Palgrave, 2010.

Cameron, Ross P. "Truthmakers and ontological commitment: or how to deal with complex objects and mathematical ontology without getting into trouble." *Philosophical Studies* 140 (2008):

Coffa, Alberto. *Foundations of Inductive Explanation*. Doctoral dissertation. Ann Arbor: University of Pittsburgh, 1973.

Colyvan, Mark. *The Indispensability of Mathematics*. Oxford: Oxford UP, 2001.

Craver, Carl F. *Explaining the Brain: Mechanisms and the Mosaic Unity of Neuroscience*. Oxford: Oxford UP, 2007.

Eddington, Sir Arthur. *The Philosophy of Physical Science*. Ann Arbor: U of Michigan, 1958.

Field, Hartry H. *Science Without Numbers: A Defense of Nominalism*. New Jersey: Princeton, 1980.

Frege, Gottlob. *Philosophical Writings*. Oxford: Basil, 1960.

French, Steven. "The Interdependence of Structure, Objects and Dependence." *Synthese* 175 (2010): 89-109.

Gödel, Kurt. "What is Cantor's Continuum Problem?" In *Philosophy of Mathematics: Selected Readings*, edited by Paul Benacerraf and Hilary Putnam, 258-273. New Jersey: Prentice-Hall, 1964.

Hodgkin A.L. and A.F. Huxley. "A Quantitative Description of Membrane Current and its Application to Conduction and Excitation in Nerve." *Journal of Physiology* 117 (1952): 500-544.

Ladyman, James and Don Ross. *Every Thing Must Go: Metaphysics Naturalized*. Oxford: Oxford UP, 2007.

Leplin, Jarrett. *A Novel Defense of Scientific Realism*. New York: Oxford, 1997.

Lyon, Aidan and Mark Colyvan. "The Explanatory Power of Phase Spaces." *Philosophia Mathematica* 16 (2008): 227-243.

Lyre, Holger. "Structural Invariants, Structural Kinds, Structural Laws."

Nerlich, Graham. "What Can Geometry Explain?" *The British Journal for the Philosophy of Science* 30 (1979): 69-83.

Pincock, Christopher. "Mathematical explanations of the rainbow." *Studies in History and Philosophy of Modern Physics* 42 (2011): 13-22.

Pincock, Christopher. *Mathematics and Scientific Representation*. Oxford: Oxford UP, 2012.

Psillos, Stathis. *Scientific Realism: How science tracks truth*. London: Routledge, 1999.

Putnam, Hilary. *Mathematics, Matter and Method: Philosophical Papers, Volume 1*. Cambridge: Cambridge UP, 1975.

Putnam, Hilary. *Philosophy of Logic*. New York: Harper, 1971.

Quine, Willard Van Orman. *From a Logical Point of View: Logico-Philosophical Essays*. New York: Harper, 1961.

Quine, W.V. *The Ways of Paradox and Other Essays*. New York: Random House, 1966.

Quine, W.V. "Whither Physical Objects?" In *Boston Studies in the Philosophy of Science: Essays in Memory of Imre Lakatos*, edited by R.S. Cohen, P.K. Feyerabend, and M.W. Wartofsky, 497-504. Dordrecht: Reidel, 1976.

Redhead, Michael. "Models in Physics." *British Journal for the Philosophy of Science* 31 (1980): 145-163.

Saatsi, Juha. "Living in Harmony: Nominalism and the Explanationist Argument for Realism." *International Studies in the Philosophy of Science* 21 (2007): 19-33.

Saatsi, Juha. "The Enhanced Indispensability Argument: Representational versus Explanatory Role of Mathematics in Science." *British Journal for the Philosophy of Science* 62 (2011): 143-154.

Shapiro, Stewart. *Philosophy of Mathematics: Structure and Ontology*. Oxford: Oxford UP, 1997.

Steiner, Mark. "Mathematics, Explanation, and Scientific Knowledge." *Nous* 12 (1978): 17-28.

Van Fraassen, B.C. *The Scientific Image*. Oxford: Oxford UP, 1980.

Weinberg, Steven. *Dreams of a Final Theory*. New York: Pantheon, 1992.

Weslake, Brad. "Explanatory Depth." *Philosophy of Science* 77 (2010): 273-294.

Wigner, Eugene. "The unreasonable effectiveness of mathematics in the natural sciences." *Communications of Pure and Applied Mathematics* 13 (1960): 1-14.

Worrall, John. "Structural Realism: The Best of Both Worlds?" *Dialectica* 43 (1989): 99-124.