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The Effects of an Experiential Learning Course on Secondary Student Achievement and Motivation in Geometry

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ABSTRACT

In 2012, the President's Council on the Advancement of Science and Technology (PCAST) predicted one million jobs in the fields of science, technology, engineering, and math (STEM) would go unfilled in the United States due to the lack of interested and qualified graduates matriculating in American universities, colleges, and technical schools (PCAST, 2012). In order to bolster interest and proficiency in STEM, research suggests instructional pedagogy incorporate experiential learning focused on solving real societal problems that are relevant to learners. Few studies have investigated the effects of such pedagogy within the context of a secondary-level, geometry course. A quantitative, quasi-experimental design was employed to determine the effect of an experiential learning course, Geometry In Construction, on secondary student achievement and motivation in geometry. Data were collected from 181 secondary students in ninth and tenth grade attending a large, suburban, Midwestern, public high school. Participants experienced a full academic year of instruction in either Geometry In Construction or a traditional geometry course. Achievement in geometry was measured using scores from a Missouri Geometry End of Course Practice Exam. Motivation to learn geometry was measured using John Keller's Course Interest Survey (Keller, 2010) based on Keller's ARCS model of motivation (Keller, 1987a). Analysis of the data indicates significantly higher achievement in geometry and motivation to learn geometry for students experiencing the Geometry in Construction curriculum. The effect is more pronounced among females. On this basis, it is recommended that geometry curricula incorporate experiential learning focused on solving real problems that are relevant to

learners. Further research is needed to determine how this instructional model could be applied to other courses in order to improve interest and preparation for STEM careers.

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DEDICATIONS

This dissertation is dedicated to my late mother Jean Iris Black who was and will always be my source of inspiration ~ Chanua

I would like to dedicate this dissertation to my lovely wife Monique, who worked equally as hard and made just as many sacrifices as I did to earn this degree. We did it, my love!

~Ted

CHAPTER 1

INTRODUCTION

The national call to address the need for high school and college students to enter postsecondary careers in science, technology, engineering, and mathematics (STEM) continues. The failure to increase the number of STEM professionals is perceived as a threat to the ability of the United States to compete in a global economy (National Academies of Sciences, Engineering, and Medicine, 2007). In a report published by The Center on Education and the Workforce at Georgetown University, Carnevale, Smith, and Strohl (2010) indicated that between 2008 and 2018, there would be an increase of one million STEM jobs, with a large percentage of those jobs requiring some form of postsecondary training. Additionally, shortages are predicted in professions that are related to STEM but traditionally viewed as non-STEM, requiring some related STEM training. Such professions include physicians, nurses, advanced manufacturing professionals, and K-12 mathematics and science teachers (President's Council on the Advancement of Science and Technology [PCAST], 2012). "To meet the goal of an additional one million STEM college graduates in the next decade, the U.S. would need to graduate an additional 100,000 per year, representing an approximately 33% increase over current production rates" (PCAST, 2012, p. 2).

Over the past decade, researchers have been addressing this perplexing shortage from a range of perspectives. Miller and Hurlock (2017) explored the issue in terms of the underrepresentation of minorities, particularly females, entering the field. Bahar and Adiguzel (2016) made comparisons between countries as they looked to discover factors influencing interest in STEM from high school students in America and Turkey. Derivative studies are emerging from the continued sense of urgency and many are aligned to findings and recommendations from PCAST. The recommendation from PCAST to transform STEM teaching and learning for K-12 students was explored within the context of an experiential geometry course called Geometry In Construction (GIC). Students taking the GIC course learn principles of geometry, career technical education, and construction through an experiential learning project in which they build a small house and donate it to a charitable organization serving the needs of homeless community members (Contextual Learning Concepts, n.d.a; Taketa, 2017).

Background

A large number of American schools still follow the factory school teaching model where students in a classroom are taught the same standards, at the same time, using the same materials and textbooks. (Darling-Hammond, 2010; Schrenko, 1994). Itin (1999) described traditional K-12 education as the teacher "being in a power position in relation to the students in terms of possessing the knowledge and the evaluation of learning" (p. 4). Concerns with this model were previously expressed by Freire (1973) who found it disturbing and unethical to have the teacher as the individual dominating the learning experience. Freire (1973) compared the traditional education process to the banking approach where the teacher deposits the information into the student so they can withdraw the information as needed. This is illustrated by students who are able to regurgitate information, but struggle to use the learned information in an application that involves higher levels of thinking. This is worth noting because recall and reproduction serve as the lowest level in terms of cognitive function and do not foster the ability to comprehend (Bloom, 1956; Piaget, 1936). Dewey (1938) viewed the educational process as a partnership involving the educator and student working together in a purposive learning experience. He linked experience with reflection, which in turn linked understanding with doing. Kolb (1984) described humans as innate learners and named a theory to formalize this process. Kolb's experiential learning theory recognized experience as the catalyst for engaging in the process of dialectic inquiry and learning, "the process whereby knowledge is created through the transformation of experience" (Kolb, 1984, p. 41). Similar to Friere, Wigginton (1986) highlighted the importance of the student role during the learning experience. He believed that the pinnacle of learning is only reached when the student is the one processing the information.

Building on the work of early philosophers, The Association for Experiential Education (AEE, n.d.) described experiential education as "a philosophy informing many methodologies in which educators purposefully engage with learners in direct experience and focused reflection" (para. 1). In such instances, the learner is "actively engaged in posing questions, investigating, experimenting, being curious, solving problems, assuming responsibility, being creative, and constructing meaning" (AEE, n.d., para. 4). The philosophies of Dewey and Friere, who expressed a concern for understanding the subject matter within an experience (experiential learning), ground this description of experiential education (AEE, n.d.). This philosophy emphasizes the importance of carefully choosing learning experiences that are relevant and meaningful to the participants. Such authentic experiences allow the learners to connect emotionally, spiritually, intellectually, and physically. The experience should be investigative by nature, permitting the learners to ask questions, experiment, take risks, and pose solutions that may or may not lead to success. Through reflection, critical analysis, and synthesis the learner is able to use the results of the experience as a basis for future learning (AEE, n.d.).

Eyler (2009) noted the significance of experiential education, as she described the impact that it has on learners often leading to the following outcomes:

- a deeper understanding of the subject matter than is possible in a classroom alone;
- the capacity for critical thinking and application of knowledge in complex to ambiguous situations;
- the ability to engage in lifelong learning, including learning in the workplace. (p. 26)

"The process by which students develop the capacity to use advanced formal reasoning processes involves confronting dissonant information and making sense of it. It requires them to monitor their own understanding and to recognize and grapple with alternative perspectives" (Eyler, 2009, p. 27). This is the essence of experiential education, since it fosters this type of intellectual thinking.

Purpose

The effects of an experiential learning curriculum on secondary student achievement in geometry and motivation to learn geometry were measured using a quasi-experimental paradigm. Achievement in geometry is defined as the degree to which students master the Missouri Learning Standards (content standards) as measured by the

Missouri Geometry End of Course Examination (Missouri Department of Elementary and Secondary Education [MO DESE], 2019a). Motivation to learn geometry is defined as the situational motivation a student has for learning the content and skills of geometry in their geometry class at a particular time as described in Keller's Attention, Relevance, Confidence, and Satisfaction (ARCS) model (Keller, 1987a). Keller's Course Interest Survey (CIS) was used to measure motivation to learn geometry and consists of four subscales: attention, relevance, confidence, and satisfaction (Keller, 2010).

Research Questions

There are many studies regarding Keller's (1987a) ARCS motivational model, however, there is a gap in the literature linking this model with experiential education in a geometry classroom. The following questions are therefore raised.

1. What effect does experiencing the Geometry In Construction curriculum have on the achievement in geometry of secondary students compared to experiencing a traditional geometry curriculum as measured by the Missouri Geometry End of Course Exam?
2. Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?
3. What effect does experiencing the Geometry In Construction curriculum have on the motivation of secondary students to learn geometry compared to experiencing a traditional geometry curriculum as measured by Keller's (2010) Course Interest Survey?

4. Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?

Hypotheses

H_0 1: There is no significant difference between the achievement in geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by Missouri Geometry End of Course Exam scores.

H_0 2: Experiencing the Geometry In Construction curriculum does not affect the achievement in geometry of secondary males and females differently as measured by Missouri Geometry End of Course Exam scores.

H_0 3: There is no significant difference between the motivation to learn geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by scores on Keller's (2010) Course Interest Survey.

H_0 4: Experiencing the Geometry In Construction curriculum does not affect the motivation of secondary males and females to learn geometry differently as measured by scores on Keller's (2010) Course Interest Survey.

Significance

In response to the disconnection between workforce expectations and what is happening in classrooms, high schools across the United States are implementing programs such as GIC (Contextual Learning Concepts, n.d.a). The GIC program design puts the philosophy of experiential education into action as students learn geometry within the context of building a small house that will be donated to a local charitable organization serving the needs of homeless community members (Taketa, 2017). The GIC curriculum combines experiential learning with a focus on applying STEM knowledge and skills to address a societal problem. In addition to concept attainment, students engage in civic citizenship, as the construction project design meets a community need. Several studies concluded that students are more motivated to learn when they are invested and care about the subject matter, which is more likely to occur in a workplace or community project setting than in a classroom (Deslauriers, Rudd, Westfall-Rudd, & Splan, 2016; Eyler, 2009; Kelley & Knowles, 2016). Adding the element of service learning, where students provide a service to their community, to experiential learning increases the relevancy of STEM coursework and leads to further increases in engagement, motivation and questioning from students as well as an improved ability to apply concepts to solve a problem (Donaghy & Saxton, 2012; Lake, Winterbottom, Ethridge, & Kelly, 2015; Tawfik, Trueman, & Lorz, 2014). However, more research is needed to measure the impact of programs like GIC on student achievement in STEM subjects and interest in STEM careers. Research in this area will: (a) support educational leaders and teachers complying with the recommendation from PCAST (2012) to transform STEM teaching practices and learning experiences by exploring innovative models for teaching and learning mathematics; (b) explore ways in which agencies can work collaboratively to design innovative learning experiences for school based programs; and (c) determine what males and females perceive to be the factors motivating them to learn geometry. Research is needed to determine to what degree the ARCS model of motivation impacts learning high school geometry.

There are a number of limitations and delimitations that should caution against overgeneralizations made from the results.

Limitations

- The nonprobability sampling method used to select the treatment and control group does not support generalizations to the larger, national population of students taking GIC courses. In order to improve statistical power, a large sample size was desirable; therefore, the sampling method had elements of a total population, purposive method in that the treatment group included all students enrolled in GIC and traditional geometry at the study site (Laerd, 2012). The sampling method also had elements of convenience sampling because the research site was chosen on the basis of its proximity and accessibility to the researchers.
- Only post-treatment measures of motivation were collected due to the timing and duration of the study; therefore, changes in motivation were not quantified for the treatment or the control group. Only comparative differences in motivation between the treatment and control group were examined.

Delimitations

- Gender was the only demographic examined as an independent variable although other factors such as socioeconomic status, race, and ethnicity may also impact the dependent variables of achievement in geometry and motivation to learn geometry. Examining the effect of gender provides an additional layer of depth to the data analysis and subsequent findings. It also contributes a unique perspective related to experiential education in a geometry course to the long debated knowledge base concerning mathematical achievement differences between males and females (Benbow & Stanley, 1980; Cimpian, Lubienski, Timmer, Makowski, & Miller, 2016; Ganley & Lubienski, 2016; Wang, Eccles, & Kenny, 2013).

- Achievement in geometry is defined narrowly by the degree of mastery of geometry content standards as measured by the Missouri Geometry End of Course Exam. Changes in specific skills such as craftsmanship, ability to design solutions, and cooperation with others, which could be seen as signs of achievement resulting from exposure to an experiential curriculum, were not assessed. When looking for differences between the treatment and control groups, it is necessary to focus on criteria, such as mastery of geometry content, that students could acquire in both GIC and traditional geometry courses.
- Motivation to learn geometry is viewed through the lens of Keller's (1987a) ARCS model, and the measure of motivation focuses only on the effect of a geometry curriculum on the self-reported scores of attention, relevance, confidence, and satisfaction. Acting on the suggestions put forth in PCAST (2012), identification of innovative pedagogy capable of enhancing interest in STEM careers was sought. Keller's (1987a) ARCS model provides a framework that specifically focuses on "increasing the motivational appeal of instruction" (p. 2) rather than a behavioral model that focuses on "changing the personalities of students" (p. 9).

Definitions of Terms

Experiential Education. Experiential education is a holistic philosophy, where carefully chosen experiences supported by reflection, critical analysis, and synthesis, require the learner to take initiative, make decisions, and be accountable for the results, through actively posing questions, investigating, experimenting, pursuing curiosity, solving problems, assuming responsibility, expressing creativity, constructing meaning, and integrating previously developed knowledge (AEE, n.d.). "Learners engage intellectually, emotionally, socially, politically, spiritually, and physically, in an uncertain environment where the learner may experience success, failure, adventure, and risk taking" (Itin, 1999, p. 6). The philosophy of experiential education allows for various expressions including service learning, cooperative learning, adventure learning, problem-based learning, and action learning (Itin, 1999).

Service Learning. Service learning is a method of teaching where academic learning experiences provide a service to the community, often fostering a sense of civic responsibility and personal growth for the student. (National Youth Leadership Council, 2018).

Achievement in Geometry. Achievement in geometry is a dependent variable representing the degree to which students master the Missouri Learning Standards (content standards) as measured by the Missouri Geometry End of Course Exam.

Motivation to Learn Geometry. Motivation to learn geometry refers to the situational interest a student has toward learning geometry. Situational motivation indicates how much a student desires to participate in classroom activities and actively pursue learning the content and skills associated with a particular class at a particular time as a result of the specific instructional practices and materials used by the teacher (Keller, 1987a). Situational motivation does not indicate the overall desire a student has for academic success in all courses or learning situations. Motivation to learn geometry is a dependent variable measured using the Course Interest Survey based on Keller's (1987a) ARCS model of motivation and consisting of four subscales representing the conditions that must be addressed to promote and sustain motivation during learning: attention, relevance, confidence, and satisfaction (Keller, 2010).

Attention. Attention is one of the four conditions that must be met to promote and sustain motivation according to Keller's (1987a) ARCS model. Attention is a subscale measure on the Course Interest Survey and is therefore a dependent variable. Keller (1987a) described attention as "a prerequisite for learning" that involves the need for students to constantly respond to stimuli that interests them in learning (p. 2).

Relevance. Relevance is one of the four conditions that must be met to promote and sustain motivation according to Keller's (1987a) ARCS model. Relevance is a subscale measure on the Course Interest Survey and is therefore a dependent variable. Keller (1987a) described relevance as a personal appreciation and connection to the learning experience rather than the end value of the learned content itself.

Confidence. Confidence is one of the four conditions that must be met to promote and sustain motivation according to Keller's (1987a) ARCS model. Confidence is a subscale measure on the Course Interest Survey and is a dependent variable. Keller (1987a) described confidence as "an expectancy for success" (p. 2).

Satisfaction. Satisfaction is one of the four conditions that must be met to promote and sustain motivation according to Keller's (1987a) ARCS model. Satisfaction is a subscale measure on the Course Interest Survey and is a dependent variable. Keller (1987a) described satisfaction as a positive feeling one might have about their personal accomplishments as a result of learning.

Geometry In Construction. Geometry In Construction is an experiential geometry curriculum originally developed in 2005 by Scott Burke, Tom Moore, and Dave Dillman to merge the curricular content of traditional geometry, construction, and career technical education in order to create a "contextualized model for teaching" (Contextual Learning Concepts, n.d.a). The GIC course studied involves students learning geometry within the context of building a small house to be donated to a local charity serving the needs of homeless community members.

Traditional Geometry. Traditional geometry is a geometry curriculum delivered by teachers employing a direct instruction approach. Students learning by this approach typically receive a daily lecture followed by in-class guided practice, sometimes in small groups, and additional homework to be completed by practicing algorithms that were taught in class.

Organization

Organization follows the traditional dissertation format. The first chapter describes the problem of one million STEM jobs going unfilled in the U.S. due to a lack of interest and qualified candidates and the threat that poses to the continuation of the U.S. as a world economic and technological leader (PCAST, 2012). In addition, chapter one situates the study within the context of the need to develop innovative, experiential instructional models that seek to enhance student motivation and achievement in STEM subjects. Geometry In Construction, an experiential education geometry curriculum, was chosen for study because there is a gap in the literature regarding the impact of experiential education in a geometry classroom on motivation as described by Keller's (1987a) ARCS model of motivational design. Chapter one also describes the purpose, research questions, significance, limitations, delimitations, and defines relevant terms. Chapter two provides a review of the literature focused on experiential education, motivation, and mathematics pedagogy as it relates to STEM while further describing Keller's ARCS model of motivational design which, along with experiential education, serves as the theoretical framework. Chapter three describes the specific methodology including the research design, instruments used to gather data, sampling methods, and procedures used for data collection and analysis. Chapter four presents the data and overall findings. Chapter five discusses the findings, conclusions, and implications for practice and further research.

CHAPTER 2

REVIEW OF LITERATURE

A quasi-experimental design was used to determine the effect of experiential learning instruction on achievement in geometry and motivation to learn geometry among secondary students. The sample consisted of high school students enrolled in geometry courses at a large, suburban, Midwestern, public high school. The treatment group received geometry instruction through an experiential learning course called Geometry in Construction. The experiential learning approach was compared to the traditional, direct-instruction method of teaching geometry. This in-depth review of literature provides key information pertaining to the variables of the study. Key descriptors used to identify preliminary sources include John Keller's ARCS Model of Motivational Design, experiential learning + STEM, socio-scientific issues, lack of interest in STEM, Geometry In Construction, self-efficacy in STEM; learning STEM + interventions, career interest in STEM, student interest in STEM, and minorities and women in STEM. Using these key descriptors, EBSCO produced 5,651 results and ERIC produced 2,557 results. The review is organized according to the guiding theoretical framework, independent variables of interest, and outcome variables of interest.

Theoretical Framework

Attention, Relevance, Confidence, and Satisfaction (ARCS) Model of Motivation

Lack of motivation is a contributing factor leading to the inability of many K-12 students to attain sufficient skills in STEM related courses. Holdren, Lander, and Varmus (2010) reported that students from multiple ethnicities, who are failing in STEM subjects, complain that the courses are too difficult and boring. Hossain and Robinson (2012) suggested that overcoming the STEM barriers will require targeted attention to education components for students from elementary through college. Onwu and Kyle (2011) described the importance of relevancy in STEM courses. They argued for ways in which educators can link classroom learning experiences to real life socio-scientific issues, which would in turn make learning more relevant and prepare students for active participation in society. Psychologist John Keller addressed this notion of relevancy and included it as a component in his creation of the ARCS model of motivation.

Origin of ARCS

Visser and Keller (1990) noted lesson design as a crucial area of focus over the past decade. Their analysis of lesson design depicts an emphasis on cognitive skills rather than the motivational requirement of the learner. Visser and Keller (1990) stated, "instruction even when prepared according to sound instructional design principles often does not stimulate students' motivation to learn" (p. 468). The formation of ARCS model of motivation integrates Keller's recognition of the correlation between instructional design and the desire of students to learn. Keller (1987b) noted the absence of a comprehensive motivational framework and explained how much of the initial work related to motivation described psychological approaches to changing motivation characteristics (McClelland, 1965) or job satisfaction and work performance (Steers & Porter, 1987). Furthermore, educators studied motivation in terms of classroom management (Doyle, 1985), reinforcement (Skinner, 1961) or affective instructional outcomes (Krathwohl, Bloom, & Masia, 1964). According to Keller (1987a), the outcomes from this work were "somewhat restricted in their approach and theoretical foundation" (p.2). The ARCS model was originally developed by John Keller in 1979 and 1983. "These models were based on the expectancy-value theory, which derives from Tolman (1932) and Lewin (1938), according that motivation is the result of satisfaction of personal needs (values) and also the amount of their expectancy to succeed (the expectancy)" (Keller, 1987a, p.2). As Keller sought to move his work from theory to practice, he posed two questions: (a) "Is there a possibility of synthesizing multiple theories of motivation into one simple model that can be used by practitioners?; and (b) Is there a systematic approach to designing motivating instruction?" (Keller, 1987a, p.2). These questions led to the systematic design of the ARCS model. Keller's work on the motivational design transitioned through several phases before he generated the useful acronym, ARCS, which highlights the central features of the design: attention, relevance, confidence, and satisfaction. The ARCS model is grounded in social learning theory and humanist psychology and takes on a system approach that integrates multiple theories (Jacobson & Xu, 2004). Because Keller's motivational work comes from a plethora of theories, some of which include the social learning theory, field theory, and self-efficacy theory, Keller considers the design to be both a theory and a macro model (Keller, 1983).

ARCS Model

"The ARCS model is a method for improving the motivational appeal of the instructional materials" (Keller, 1987b, p.2). The ARCS model is represented by the following distinctive features:

- four conceptual categories that incorporate concepts and variables that characterize human behavior;

- strategies used to enhance motivation; and
- a systematic design process (motivational design) that can be used with other traditional instructional design models (Keller, 1987b, p.2).

"Motivational design refers to the process of arranging resources and procedures to bring about changes in motivation" (Keller, n.d.b, para. 1). According to Keller (n.d.a), "one of the goals of motivational design is to prepare a set of motivational tactics that are in alignment with learners' motivational needs and are complimentary with the overall instructional plan" (para. 1). Additionally, Keller (n.d.b) highlighted the importance of such motivational tactics directly supporting instructional goals. He stated, "Sometimes the motivational features can be fun or even entertaining, but unless they engage the learner in the instructional purpose and content, they will not promote learning" (Keller, n.d.b, para. 5). "While there are many elements in a course that could affect motivation, such as the behaviors of the teacher, structure of the lessons, materials used, and course structure, ARCS model offers assistance in specific areas" (Keller, n.d.a, para. 2). The model is comprised of four components that must be present for individuals to initiate and sustain motivation: attention, relevance, confidence, and satisfaction. Malik (2014) described that Keller's model "raises the attention of students during instruction, develops a relevance to the students' requirements, creates a positive expectation for success and supports student satisfaction by reinforcing success" (p.194). Figure 1 illustrates the three essential elements of each category of the ARCS model (Keller, 1987a; Shellnut 1996).

Attention comprises perceptual arousal, inquiry arousal, and variability (Keller, 1987a). Perceptual arousal includes strategies for gaining and sustaining the interest of students over time. Inquiry arousal describes techniques used to provoke thinking, such as the use of problem-solving. Last, variability references the use of varied instructional approaches. Examples of this might include, lecture, group activities, games, visuals, and technology (Shellnut, 1996).

ARCS Category and Essential Elements	Guiding Questions to Help Address Essential Element
Attention	
Perceptual arousal	What can I do to capture their interest?
Inquiry arousal	How can I stimulate an attitude of inquiry?
Variability	How can I maintain their attention?
Relevance	
Goal orientation	How can I best meet my learner's needs? (Do I know their needs?)
Motive matching	How and when can I provide my learners with appropriate choices, responsibilities, and influences?
Familiarity	How can I tie the instruction to the learner's experiences?
Confidence	
Learning requirements	How can I assist in building a positive expectation for success?
Success opportunities	How will the learning experience support or enhance the student's belief in their competence?
Personal control	How will the learners clearly know their success is based on their efforts and abilities?
Satisfaction	
Natural consequences	How can I provide meaningful opportunities for learners to use their newly acquired knowledge/skill?
Positive consequences	What will provide reinforcement to the learner's successes?
Equity	How can I assist the student in anchoring a positive feeling about their accomplishments?

Figure 1. The three essential elements of each motivational category of the ARCS Model. Adapted from Keller, J. M. (1987b). The Systematic Process of Motivational Design. *Performance and Instruction, 26*(9-10), 2.

Relevance can be described as the ability of learners to connect the content with their personal needs and desires (Keller, 1987a). Keller (1987a) delineates three elements used to address personal needs and wants of learners: goal orientation, learner choice, and familiarity. Goal orientation describes outcomes that are derivative of the learning such as obtaining a job. Learner choice or what Keller (1987a) called motive matching involves the decision of the learner to select specific learning strategies. One example of this is choosing to work independently or in a group. Finally, familiarity is the ability of learners to connect preexisting knowledge or personal experiences to the content being learned (Keller, 1987a).

Confidence provides a sense of belief that one can accomplish given tasks (Keller, 1987a). In building confidence, Keller (1987a) and Shellnut (1996) noted that learning should be tied to clear objectives, success opportunities should be provided early and often, and personal control of learning should be made available through options.

Satisfaction suggests that learning must lead to gratification (Keller, 2000). The following elements increase learner satisfaction: connecting learning to real-world experiences, simulations, or projects; providing both intrinsic and extrinsic rewards; and assuring that reward matches achievement (Keller, 1987a).

Two motivational instruments for assessing the motivational quality of instructional situations accompany the ARCS model (Small, 1997). The Instructional Materials Motivation Survey (IMMS) asks students to rate ARCS-related statements in relation to the instructional materials that were used, whether within a classroom, stand-alone print material, or online (Keller, 2010). The Course Interest Survey (CIS) measures the reaction of students to instructional materials and methods, whether face-to-face or online (Keller, 2010).

Related Research

ARCS model is one of the most popular motivational designs that has been grouped with various instructional pedagogy, particularly those consisting of problem-based learning as an instructional approach (Carliner, Ribiero, & Boyd, 2008). In an extensive literature review of empirical studies, Li and Keller (2018) summarized research on applying ARCS model in a variety of educational settings around the world. Li and Keller (2018) studied 27 peer-reviewed journals in which they shared various ways in which ARCS model had been applied. "Most studies included strategies for all four factors in the ARCS model" (Li & Keller, 2018, p. 60). Overall results from this review of studies showed: (a) ARCS is a flexible model that can be used in a variety of environments; (b) quantitative methods are most often used in research involving ARCS due to the clear guide-lines the model has, thus making it easy for researchers to examine the effects of the model on motivation; (c) students showed positive attitudes toward ARCS strategies even as they were implemented in varied educational settings; (d) variables in the cognitive domain were inconsistent, some indicating cognitive gains while others reporting no differences; (e) learner behaviors varied in that some students improved time on task while others showed no difference; (f) retention/completion rates were reported as improved; and (g) there was no clear indication of ARCS model affecting psychological traits of students, but researchers considered a connection between intrinsic feelings of some subscales affecting the motivation and cognition of learners (Li & Keller, 2018).

Motivation

Motivational Constructs Influencing STEM

Motivation has been studied over the years by psychologists in an attempt to define and explain what intrinsically makes us take action toward achieving the simplest to the most complex tasks. Several studies link motivation with desire to learn. Desire to learn was noted by Lazowski and Hullemann (2016) as an important catalyst for increasing student achievement in science. Miller and Hurlock (2017) honed in on the noticeable differences between males and females who are motivated to pursue STEM, and supporting studies describe this gap to be both “a progressive and persisting problem” (Cronin & Roger, 1999, p.637). According to Wang, et al. (2013), it is not the lack of skill but the lack of interest and motivation that serve as the primary factor behind the absence of professionals, including low rates of females, in certain STEM fields. Various motivational constructs are seen as essential components and catalysts for creating the desire to seek STEM learning K-12 and beyond (National Academies of Sciences, Engineering, and Medicine, 2019). The implementation of motivational constructs affects a number of crucial decisions made by students. One important decision includes the selection of courses leading toward specific career paths (Musu-Gillette, Wigfield, Harring, & Eccles, 2015).

In an effort to include the appropriate motivational constructs, researchers have investigated student perceptions regarding STEM, as they are often barriers to motivation. There are specific interventions or targeted motivational constructs developed to assist students in overcoming some of the common misconceptions they have about STEM, including stereotypes that exclude groups from believing they are capable of participating (National Academies of Sciences, Engineering, and Medicine, 2019). As illustrated in Figure 2, educators have relied on the following existing

Motivational Theories	**Description**	**Researchers**
Expectancy-Value Theory	Two factors motivate individuals to achieve: • expectancies for success • value for the task	John Atkinson Jacquelynne Eccles
Attribution Theory	Individuals are motivated by explanations or causes that can be contributed to their success.	Bernard Weiner
Social Cognitive Career Theory	Career interest and development are driven by an individual's self-efficacy, outcome expectations, and goals.	Robert Lent Steven Brown Bail Hackett
Social Cognitive Theory	Individual are influenced by the behaviors of others. Often those actions are replicated and guide subsequent behaviors.	Albert Bandura
Goal Orientation Theory	Learners approach situations with the goal of mastering new skills or outperforming their peers.	Carol Dweck
Self-determination Theory	Individuals are intrinsically motivated by three distinctive psychological needs which allow them to grow and change (competence, relatedness, and autonomy).	Edward Deci Richard Ryan

Figure 2. Contemporary theories of motivation.

contemporary theories of motivation to support targeted interventions: expectancy-value, attribution, social cognitive, goal orientation, and self-determination (Cook & Artino, 2016; Schunk, Pintrich, & Meece, 2008). The following section of the literature review provides an explanation of several motivational constructs and connects them to research targeting STEM motivation.

Expectancy-value theory. The expectancy-value theory (Eccles et al., 1983) explains that "students' beliefs concerning the degree to which they are confident in accomplishing an academic task (self-efficacy) and the degree to which they believe that the academic task is worth pursuing (task value) are two key components for understanding students' achievement behaviors and academic outcomes" (Liem, Lau, & Nie, 2008, p.488). In context to STEM learning, there is empirical data linking performance of students in various STEM subjects to their positive expectancy toward success (Schunk, Pintrich, & Meece, 2008). Additionally, such students are predicted to pursue STEM learning if it is aligned with their personal needs (value) (Eccles et al., 1983). Evidence for this relationship exists in studies where student motivation and performance increase as a result of interventions targeting "value". For example, Acee and Weinstein (2010) and Hulleman and Harackiewicz (2009) conducted studies grounded in the expectancy value theory. While each study focused on interventions targeted at increasing the utility value of math and science for students, very different approaches were taken. Acee and Weinstein (2010) targeted parents who they speculated would transfer their perceived value for math and science to their children. Over a two-year span, resources and materials were mailed to parents to support conversations with their children regarding the significance of math and science coursework. By the conclusion of high school, the students whose parents were in the treatment group showed higher utility value for math and science than the students in the control group. Similar results were reported by Hulleman and Harackiewicz (2009) whose study targeted an instructional intervention also aimed at increasing utility value for math and science. In their study, high school science students were randomly chosen to write essays explaining the relevancy of the content learned to their personal lives. Not only did this intervention improve the utility value of math and science for participants, but it also increased their interest and resulted in higher grades compared to the control group.

Attribution theory. The attribution theory is one way in which educators seek to develop intrinsic motivation of students who participate in STEM curricula. The attribution theory (Weiner, 1985) is described as “the explanations that students generate to understand what causes a particular success or failure experience and how experiences drive students’ motivation and behavior on future tasks” (Rosenzweig & Wigfield, 2016, p. 152). Ziegler and Heller (2000) conducted a study with physics teachers who received attribution training. Over a period of one year, students in the treatment group were given feedback connected to their work. The students were consistently told that their success on physics assignments was directly related to their effort. It was found that the students who underwent the attribution intervention demonstrated increases in their internal belief that success is attributed to effort. A similar study by Ziegler and Stoeger (2004) found an increase in intrinsic motivation among high school female chemistry students who received attribution training through informational videos. Although these studies yield promising results, Rosenzweig and Wigfield (2016) addressed major limitations noting that there were a limited number of attribution-based interventions and differing retraining paradigms affecting the interpretations of results. It was suggested that future studies assess the effectiveness of attribution retraining in STEM areas.

Social cognitive theories. The social cognitive career theory (SCCT) (Lent, Brown, & Hackett, 1994) is a motivation construct also used to determine factors motivating interest in STEM. "The SCCT was developed to examine the manner in which people develop and elaborate on career and academic interests, select and pursue choices based on interests, and perform and persist in their occupational and educational pursuits" (Soldner, Rowan-Kenyon, Inkelas, Garvey, & Robbins, 2012, p. 314). The SCCT identifies self-efficacy, outcome expectations, and goal orientation as the leading motivational constructs for shaping career choice decisions (Bahar & Adiguzel, 2016). Prior studies have identified other important factors linked to what the social cognitive theory (Bandura, 2013) names as personal and proxy agency. Personal agency references the events in life that are influenced by personal actions, such as self-motivation or self-efficacy. Proxy agency takes on the notion that people rely on others in their surroundings to help them achieve their desired outcomes. In several studies, proxy agents were identified as parents, teachers, and school-related factors which were all considered constructs for motivating learning in science (Breakwell & Robertson, 2001; Olitsky, Loman, Gardner, & Billup, 2010; Sjaastad, 2012).

Goal orientation theory. The theory of goal orientation (Dweck, 1986) also provides a framework to study how learners are motivated. "Goal orientations refer to the reasons why students pursue achievement outcomes" (Rosenzweig & Wigfield, 2016, p. 153). The theory addresses two major goal orientations, performance and mastery, in which students focus on outperforming their peers (performance) or deepening their learning of skills (mastery) (Rosenzweig & Wigfield, 2016). Mastery goals are thought to have a long term effect on the retention of knowledge and achievement outcomes (Maehr & Zusho, 2009). The research of Blackwell, Trzesniewski, and Dweck (2007) connected learner success in science to confidence in their ability to engage in science, longevity of understanding science content, and positive attitudes toward science. In a subsequent study, Fortus and Vedder-Weiss (2014) found that successful students were more likely to engage in STEM learning outside of school.

Self-determination theory. Deci and Ryan developed the self-determination theory which explores the effects of extrinsic and intrinsic motivation on human behavior (Deci, Koestner, & Ryan, 1999). Intrinsic motivation, when a learner performs an activity for personal gratification, has been a major approach to intervention in the area of STEM. In some aspects, the theory connects to components of the expectancy value theory as interventions "attempt to increase students' sense of value or connection to science and engineering" (National Academies of Sciences, Engineering, and Medicine, 2019, p. 3-9). Successful intervention strategies include those that improve student perceptions of STEM professions which subsequently improves student perceptions of the value of science content taught in schools. Role models inspire learners and enable them to see themselves in STEM professions, and this increases student engagement and achievement in STEM courses (Stout, Dasgupta, Hunsinger, & McManus, 2011).

The theories described have a harmonic relationship in that they are complimentary to one another. This is supported by the research of Rosenzweig and Wigfield (2016) who assessed the effects of these motivational constructs and grouped them under common themes. In this study, fifty-three intervention studies were reviewed and categorized according to: (a) “competence-related beliefs (e.g., self-efficacy, self-concept, confidence, and outcome expectations); (b) beliefs about values, interests, or intrinsic motivation; (c) attributions about academic success and failure; (d) beliefs about intelligence; and (e) achievement goal orientations” (Rosenzweig & Wigfield, 2016, p.149). In summary, forty-one of the studies reported significant improvements in at least one of the motivational constructs where academic outcomes were measured. Grades, quizzes, and test scores were among the areas of improvement. The studies concluded “motivation interventions do, in some circumstances, improve STEM students’ competence-related beliefs, values, interests, attributions, and beliefs about intelligence, as well as their academic outcomes such as exam performance, course taking, and more proximal outcomes such as accessing supplemental materials for their class” (Rosenzweig & Wigfield, 2016, p.155).

Motivating STEM Learners through Experience– Constructivism

John Dewey held a strong belief in democracy that helped establish the framework for his learning theories which are evident in current, 21st century learning experiences (Lake et al., 2015). Dewey (2008) posited that learners contribute to and change society by means of tangible learning experiences and activities that foster societal awareness. He asserted that learning does not just entail the simple acquisition of content knowledge but should also contribute to the larger society. Experiential education is one way in which educators have applied Dewey’s theories. This methodology encompasses a wide range of applications; however, this section of the literature review will focus on experiential education in the context of STEM.

The main types of STEM experiences found in the literature focus on service learning and problem-based learning. These experiences were applied either throughout a semester, a year-long course, or during a summer camp. In all instances, the learning experiences sought to enhance the relevancy of the material. Service learning experiences often include a component of civic responsibility and allow students to apply their learning toward addressing socio-scientific issues affecting their community (National Service Learning Clearinghouse, 2013). When applying experiential learning and socio-scientific issues in a summer camp format, several studies found students are motivated to learn by hands-on activities, understand the importance of STEM subjects, and display more interest in STEM careers (Bhattacharyya, Mead, & Nathaniel, 2011; Campbell, Lee, Kwon, & Park, 2012; Hayden, Ouyang, Scinski, Olszewski, & Bielefeldt, 2011; Mohr-Schroeder et al., 2014). However, in terms of STEM achievement, Nugent, Barker, Grandgenett, and Adamchuck (2010) suggested that longer interventions such as full-year courses are more successful for learning content than shorter interventions such as summer camps. Nugent et al. (2010) noted that students engaging in short-term interventions had a larger improvement in their attitude toward STEM compared to students participating in long-term interventions. Bhattacharyya et al. (2011) found additional measures enhance the effectiveness of summer camp interventions. For example, initiating interventions before age 11 had the largest impact on developing interest in STEM careers (Bhattacharyya et al., 2011).

STEM learning experiences focused on solving relevant, real world problems, often referred to as problem-based-learning (PBL) increase student motivation to learn and interest in STEM (Christensen, Knezek, & Tyler-Wood, 2015; Nugent et al., 2010;

Scogin, Kruger, Jekkals, & Steinfeldt, 2017; Tawfik et al., 2014). Christensen et al. (2015) found that dispositions toward STEM content and careers by middle school students participating in hands-on STEM engagement activities such as PBL, were more similar to students attending specialized STEM academy schools and professionals working in STEM fields than to their typical age-equivalent peers. This effect does lessen, though, as students get older and when the activity is required by the whole class rather than chosen by the learner (Christensen et al., 2015). Studies suggesting that PBL increases standardized test scores were not found, but Scogin et al. (2017) reported that standardized test scores did not decrease either in classrooms engaging in PBL. There were other significant improvements associated with PBL in non-cognitive skills such as enjoyment of school, confidence, and collaboration (Donaghy & Saxton, 2012; Lake et al., 2015; Scogin et al., 2017; Tawfik et al., 2014). Several studies suggested adding the element of service learning, where students engage in meaningful community improvement, to problem-based learning (Donaghy & Saxton, 2012; Lake et al., 2015; Tawfik et al., 2014). They argued that this increases the relevancy of STEM coursework which leads to higher levels of student engagement, motivation, questioning, and problem solving.

Many STEM education programs focus on experiential or service learning but not always merged together. Perhaps doing so could help strengthen programs and make them more interesting to students. Deslauriers et al. (2016) encouraged education researchers to seek “knowledge of how educational theory is applied in a program as it provides insight as to how those applications can be modified to further strengthen experiential programs” (p. 311). The effects of experiential education and Keller’s

(1987a) ARCS model of motivational design were measured when applied in an experiential geometry course, Geometry In Construction. More research is needed to measure the effect of curricula like Geometry In Construction, which combines experiential learning (building a house) with a socio-scientific issue (homelessness), on student achievement and motivation in STEM.

Mathematics Pedagogy and Achievement

Success in modern, technology-based societies often requires that citizens develop solid mathematical understanding and reasoning ability. Many careers demand critical analyses of data and application of sophisticated computations to solve problems. According to the National Research Council (NRC) (2001), “Failure to reason mathematically deprives individuals of opportunity and competence in everyday tasks, therefore, all young Americans must learn to think mathematically, and they must think mathematically to learn” (p. 16). Similarly, Sherman, Richardson, and Yard (2013) noted “A lack of sufficient mathematical skill and understanding affects one’s ability to make critically important educational, life, and career decisions” (p. 5). These studies suggest that learning mathematics is integral to personal, career, and educational success. In addition, the PCAST (2012) report, which predicted a shortage of one million STEM professionals in the U.S. over the next decade, listed difficulty with math as one of the main reasons why STEM students switch to different majors. Since mathematics is one of the four STEM disciplines and also a critical component of each of the other three, it seems logical that insufficient mathematical skill and understanding could be a factor preventing entry into STEM careers. Several studies suggest the root cause of insufficient mathematical skill and understanding is often a lack of interest and motivation for learning mathematics rather than a lack of ability (Lazowski & Hulleman, 2016; National Council of Teachers of Mathematics [NCTM], 2000; NRC, 2001; Wang et al., 2013). For the U.S. to continue as an economic and technological world leader, educators must develop innovative instructional practices that will increase student interest and motivation to learn mathematics in order that there are greater numbers of graduates prepared to succeed in STEM careers (NRC, 2001). To better understand the impact that innovative mathematics instruction may have on the achievement and motivation of diverse learners in schools today, it is helpful to review the literature on mathematics instructional practices in the U.S.

Instructional Practices in U.S. Mathematics Classrooms

There is a gap in mathematics education literature regarding specific interactions between teachers, students, and instructional materials within the classroom that will sustain the engagement needed to learn and apply mathematics principles (NRC, 2001). Several studies describe the persistence of a recitation model of instruction stemming from the early 1900’s as the dominant instructional practice in U.S. mathematics classrooms today.

Fey (1979) noted the most common form of mathematics instruction in U.S. K-12 classrooms involves a cycle of teacher-directed instruction that includes rule explanations and example solutions, followed by guided paper-and-pencil practice of textbook problems under the direct supervision of the teacher, and even more problems assigned as homework. Surveys of U.S. teachers continue to reveal most instructional time is spent presenting and practicing material from textbooks (Grouws & Smith, 2000). Additional studies report that elementary and middle school students receive most of their mathematics instruction on facts and skills while only about half receive instruction on reasoning (NRC, 2001).

The Trends in International Mathematics and Sciences Study (TIMSS) Video Study, conducted in 1995, provided hard evidence of teaching practices throughout the world as it documented with video the activities and interactions of students and teachers inside classrooms. The evidence shows little variation among mathematics instruction within the U.S. but significant differences between the U.S. and countries like Japan and Germany (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). The videos from classrooms in the U.S. show a repetitive daily pattern of instruction that starts with a short warm up problem or homework review followed by the teacher explaining and solving an example problem while asking low level questions of students (Stigler et al., 1999). Next, students solve practice problems like the one demonstrated, check answers, and receive more problems for homework (Stigler et al., 1999). The 1995 TIMSS video study showed that mathematics teachers in the U.S. are still teaching in a basic recitation format like they had been taught, with almost identical approaches as those reported in studies and surveys by the National Science Foundation and the National Advisory Committee on Mathematical Education in the 1970's (NRC, 2001). In contrast to U.S. instructional practices, videos from Germany show teachers emphasizing advanced and alternative solutions to more complex problems (Stigler et al., 1999). Mathematics teachers in Japan are seen in the video devoting extended periods of time to individual and group work for problem solving that is presented to the class (Stigler et al., 1999).

More recent studies of geometry instruction describe the integration of technology into the standard recitation pedagogical model, but the literature is sparse with regard to innovative or experiential curricula that dramatically alter the traditional recitation approach to teaching geometry. Multiple studies found Dynamic Geometry Environments, a specific type of gaming experience in which students discover geometry theorems using three-dimensional representations, improves understanding of geometry principles and enhances reasoning skills (Crompton, Grant, & Shraim, 2018; Luz & Soldano, 2018). In a study of ninth and tenth grade geometry students engaging with Dynamic Geometry Environments, Luz and Soldano (2018) identified the dialogue between students and the practice of defending arguments while answering questions and solving problems as contributing factors that improve understanding of geometry. This finding supports the inclusion of experiential and service learning in geometry curricula because discussion and argumentation are frequent practices students use in these instructional approaches while working in teams to solve authentic problems. Flipped classrooms, in which online videos and digital manipulatives enable teachers to interchange the typical sequence and location of homework and lecture, are another practice emerging in geometry classrooms (de Araujo, Otten, & Birisci, 2017). De Araujo et al. (2017) studied four secondary mathematics classrooms to categorize and evaluate the quality of flipped lessons. They classified flipped lessons as higher quality lessons with more potential for learning if the lesson required students to interact with videos and utilize digital manipulatives to make predictions and justify solutions (de Araujo et al., 2017). Regardless of the mathematics curricula, instructional practices, and reform efforts employed, results from standardized mathematics exams (National Center for Educational Statistics [NCES], 2019b; U.S. Department of Education, Institute of Education Sciences, 2019) suggest current teaching practices in the U.S. remain largely ineffective for helping students develop the deep understanding and problem solving skills necessary for success in STEM. Further research is needed on geometry instruction that incorporates active learning environments, like those created by experiential and service learning curricula, in order to measure the impact on student engagement, achievement, and motivation to learn.

Current State of Achievement in Mathematics in the U.S.

Recent mathematics standardized test data suggest a few areas of improvement, but overall the data indicate current pedagogical practices and standards-based reform initiatives have had little impact on achievement in mathematics. One indicator of the achievement levels of U.S. students is the National Assessment of Educational Progress (NAEP). The NAEP is a "congressionally mandated project administered by the National Center for Education Statistics within the U.S. Department of Education and the Institute of Education Sciences" that provides testing and data services for several subject areas including mathematics (NCES, 2019a, para. 2). The NAEP Mathematics Test measures student ability in "numeracy, measurement, geometry, algebra, data analysis, statistics, and probability" (NCES, 2019c, para. 4). The 2017 NAEP Mathematics Test data indicate 40% of fourth grade students and 34% of eighth grade students scored at or above the proficient category compared to 13% and 15%, respectively in 1990 (U.S. Department of Education, Institute of Education Sciences, 2019). These results indicate statistically significant improvements from 1990 to 2007 followed by a flat trend from 2007 to 2013 and lower scores after 2013 as shown in Figure 3 (U.S. Department of

Education, Institute of Education Sciences, 2019). Figure 4 shows a similar trend in the overall average scores on the NAEP Mathematics Test.

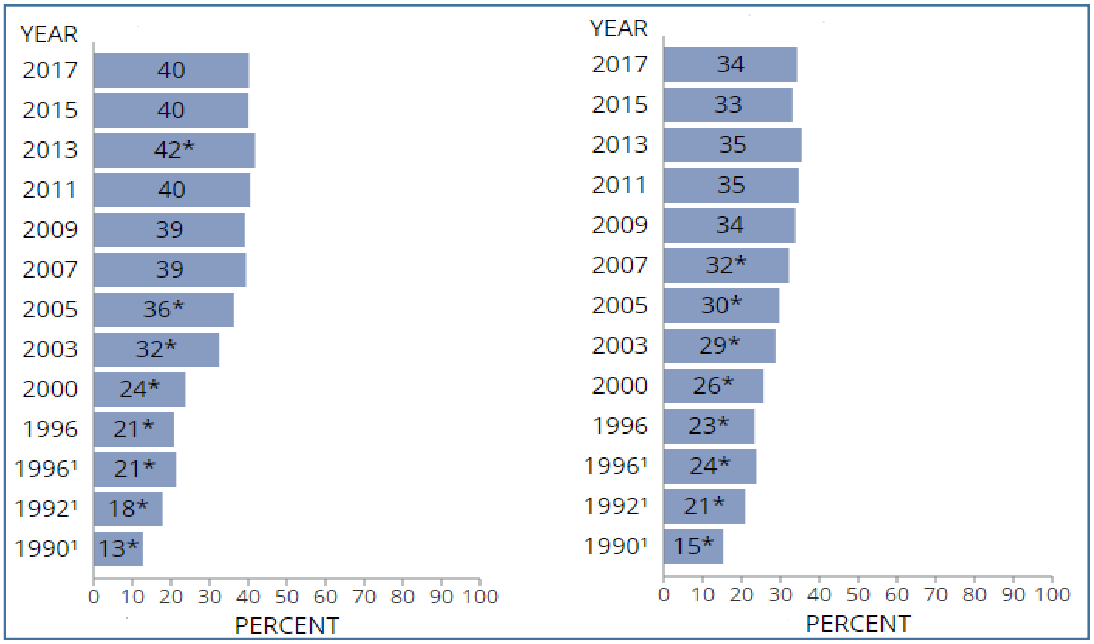

Figure 3. Percentage of fourth grade students (left) and eighth grade students (right) scoring at or above proficient on the NAEP Mathematics Test.
[1]Testing accommodations were not permitted until 1996. * indicates significantly different (p<.05) from 2017 (U.S. Department of Education, Institute of Education Sciences, 2019)

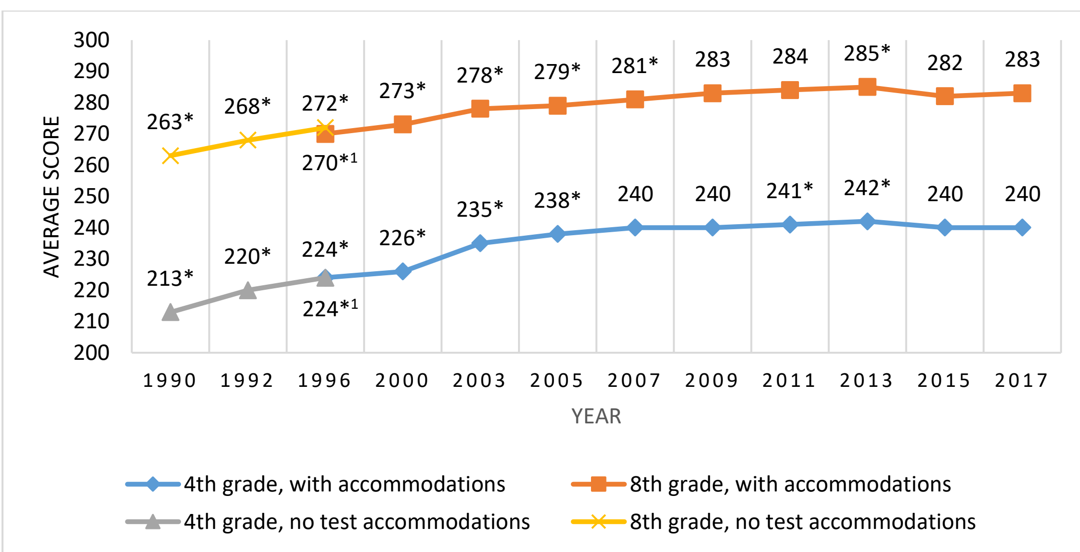

Figure 4. Average scores of fourth grade students (bottom) and eighth grade students (top) on the NAEP Mathematics Test.
[1]Testing accommodations were not permitted until 1996. * indicates significantly different (p<.05) from 2017 (adapted from U.S. Department of Education, Institute of Education Sciences, 2019)

The 2015 NAEP Mathematics Test data indicate 25% of twelfth grade students scored at or above proficient, but this number, as well as the 2015 overall average score, does not differ significantly from results recorded in 2005 (U.S. Department of Education, Institute of Education Sciences, 2019). More research is needed to determine the precise cause of these flat trends for secondary students, but they could indicate a lack of relevancy of curricula and a loss of interest and motivation for learning mathematics through traditional, teacher-directed pedagogies.

The results of international mathematics assessments such as the Program for International Student Assessment (PISA) and the Trends in International Mathematics and Science Study (TIMSS) also offer valuable data to evaluate and compare the current state of achievement in mathematics for U.S. students to other educational systems in other countries. Results from the 2015 PISA, measuring the mathematics literacy of 15-year-old students, reveal the average U.S. student score of 470 was below the overall average score of 490, thus ranking the U.S. educational system 37th out of 69 educational systems participating in the assessment (NCES, 2019b). The percentage of 15-year-old students in the U.S. scoring below level 2, which is considered baseline proficiency on a seven-level scale, was 29% (NCES, 2019b). In addition, the PISA mathematics literacy average scores for the U.S. in 2015 were significantly lower ($p<.05$) than the average scores from 2009 and 2012 as shown in Figure 5 (NCES, 2019b). The TIMSS is administered every four years to fourth and eighth grade students, and the TIMSS advanced is administered every four years to students in their final year of secondary school (NCES, 2019b). The TIMSS mathematics average scores of fourth and eighth grade students in the U.S. have improved since 1995 with the notable exception of fourth grade students in 2015 as shown in Figure 6 (NCES, 2019b). However, students scoring

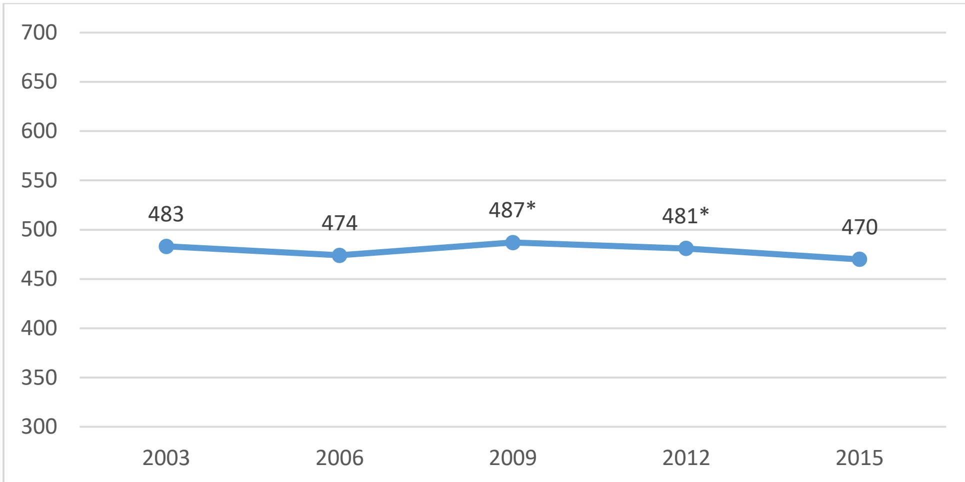

Figure 5. Average scores of U.S. 15-year-old students on the PISA mathematics literacy scale.
* indicates significantly different (p<.05) from 2015. Adapted from National Center for Educational Statistics. (2019b). Surveys and Programs [website].

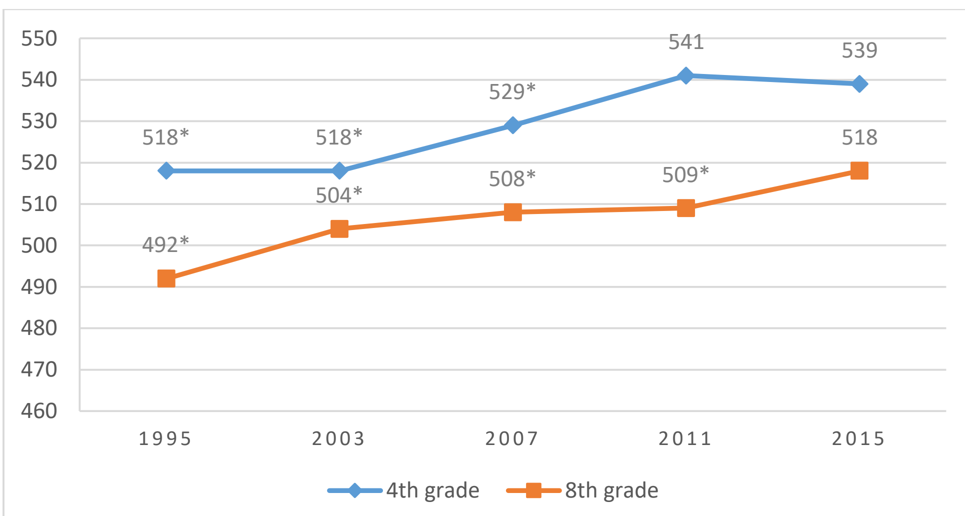

Figure 6. Average scores of U.S. fourth (top line) and eighth (bottom line) grade students on the TIMSS Mathematics Test.
* indicates significantly different (p<.05) from 2015. Adapted from National Center for Educational Statistics. (2019b). Surveys and Programs [website].

in the lowest 10th percentile did not improve significantly between 1995 and 2015 (NCES, 2019b). In 2015, the TIMSS mathematics average score of U.S. fourth grade students was higher than 34 other educational systems and lower than 10 other systems while the average score of eighth grade students was higher than 24 other educational systems and lower than 8 other systems (NCES, 2019b). Scores from TIMSS are categorized into four benchmarks: low, intermediate, high, and advanced. In 2015, 79% of U.S. fourth grade students and 70% of U.S. eighth grade students reached the intermediate or higher benchmark on the TIMSS Mathematics Test (NCES, 2019b). In 2015, the U.S. average score on the TIMSS Advanced Mathematics Test was higher than five other education systems and lower than two others, however this average score was not significantly different from the average score obtained in 1995 (NCES, 2019b).

Data from these standardized mathematics achievement tests show mixed results, but they all indicate the need to keep innovating, reforming, and improving mathematics teaching and learning in the U.S. Meta-analyses of recent research related to mathematics teaching and learning by the NRC and the National Mathematics Advisory Panel (NMAP), confirm what the NAEP and TIMSS results indicate which is that U.S. students have made some progress with basic computations, but still lag behind in their ability to think critically and apply mathematics to solve problems (NCES, 2019b; NMAP, 2008; NRC, 2001; U.S. Department of Education, Institute of Education Sciences, 2019). Low PISA scores and poor rankings on international comparisons today suggest the NRC (2001) description of U.S. mathematics curricula as “shallow and not challenging” (p. 4) is still accurate in many regions of the country. Even where mathematics curricula were dramatically improved to align with the recommendations of the NRC, NMAP, NCTM, and Common Core State Standards Initiative, what has not changed enough are the activities and interactions students engage in within mathematics classrooms.

Research-Based Mathematics Pedagogy

Several research studies agree that an effective instructional approach combines both teacher-directed and student-centered instruction with a balance among rote memorization of rules, practice of computational skills, deeper understanding of mathematics principles, and development of problem solving skills (Bruner, 1977; Larson & Kanold, 2016; NCTM, 2000; NMAP, 2008; NRC, 2001, Sherman et al., 2013;). Jerome Bruner, a cognitive psychologist at Harvard, believed “long term understanding and skill achievement are established together when students successively build upon concepts in a guided discovery process” (as cited in Sherman et al., 2013, p. 7). The recommendations from the meta-analysis of mathematics teaching and learning research conducted by the NRC in 2001, incorporate Bruner’s (1977) concept of the interdependence of skills and understanding into five interdependent strands involved in developing “mathematical proficiency” which they proposed as the essence of successful mathematics learning (NRC, 2001, p. 5). The five strands of mathematical proficiency as explained in the NRC report include:

- conceptual understanding - comprehension of mathematical concepts, operations, and relations;
- procedural fluency - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;

- strategic competence - ability to formulate, represent, and solve mathematical problems;
- adaptive reasoning - capacity for logical thought, reflection, explanation, and justification; and
- productive disposition - habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 5).

The development of mathematical proficiency means students can understand basic concepts and procedures, carry out computations, solve problems, reflect on and justify solutions, appreciate the usefulness of mathematics, and believe in their ability to learn mathematics (NMAP, 2008; NRC, 2001). The NRC established mathematical proficiency as a benchmark goal for mathematics teaching and learning with support from their meta-analysis of thousands of studies related to mathematics instruction (NRC, 2001). The NRC (2001) acknowledged that students attain mathematics proficiency through various instructional approaches, but the more effective approaches address multiple strands of proficiency simultaneously.

Several studies indicated problem solving as an effective way to engage students and help them develop multiple strands of proficiency, provided the problems are engaging to students so they value learning and see how it applies to their daily lives (National Academies of Sciences, Engineering, and Medicine, 2019; NMAP, 2008; NRC, 2001; Sherman et al., 2013). In *Principles and Standards for School Mathematics*, the NCTM (2000) described problem solving as, "engaging in a task for which the solution method is not known in advance" and noted "students must draw on their knowledge and often develop new mathematical understanding" (p. 52). Sherman et al. (2013) suggested, "solving problems should be seen not only as the purpose of mathematics but also the means by which it is learned" (p. 199). The NRC (2001) agreed, "Problem solving should be the site in which all of the strands of mathematics proficiency converge. It should provide opportunities for students to weave together the strands of proficiency and for teachers to assess students' performance on all of the strands" (pp. 420-421). Problem solving impacts the development of mathematical proficiency more when students have "frequent, extended blocks of time to work in small, cooperative groups that struggle with challenging problems and reflect on their thinking" (NCTM, 2000, p. 52). Therefore, a significant amount of instructional time should be provided for students to develop concepts and procedures they can use to solve problems (NCTM, 2000; NRC, 2001; Sherman et al., 2013). Several studies agree that students will value challenging problem-solving activities they are interested in and perceive as relevant to their personal lives or connected to their prior experiences (Keller, 1987a; NCTM, 2010; NRC, 2001; Sherman et al., 2013). Good problems and learning activities that emphasize the relevance of mathematics to everyday life can motivate students to participate in class more, express their reasoning better, and lead to higher levels of mathematical achievement (NCTM, 2000; Sherman et al., 2013). Collectively, these studies illustrate how problem solving tasks should be leveraged in mathematics curricula to enhance relevancy, student engagement, and the development of mathematical proficiency.

The use of manipulatives such as base ten blocks and fraction tiles is also highlighted in the literature as a way to strengthen concept development when teachers are intentional in connecting them to mathematical concepts and symbols (NRC, 2001;

Sherman et al., 2013). In a videotaped, ethnographic study of third-grade students using manipulatives to improve numeral proficiency, Ball (1992) demonstrated the in-depth interactions required between teachers and students in order to make manipulatives effective as instructional tools. Teachers must intentionally incorporate discussion into lessons to highlight student reasoning and provide ample time for students to explore the manipulatives and allow for multiple ways of using them to represent concepts, ideas, and thinking (Ball, 1992).

The context of building a small house, as in the GIC course, exemplifies a curriculum that incorporates many of these research-based suggestions for how mathematics should be taught through a single, year-long experience for students (Contextual Learning Concepts, n.d.a). In the process of building a house, students taking GIC work on developing multiple strands of mathematical proficiency simultaneously which, according to the NRC (2001) report, "enhances student learning about space and measure and shows promise for helping students learn about data and chance" (p. 8). This is valuable because many current and future STEM jobs require adept skills in managing data and using statistics to inform decision making. As students design, build, test, and modify structures in GIC, they also use manipulatives to solidify geometry concepts and principles (Contextual Learning Concepts, n.d.a). Constructing a house provides a concrete setting in which students must apply principles of geometry to solve the problems they encounter daily. The NRC (2001) reported, "students develop higher levels of mathematical proficiency when they have opportunities to use mathematics to solve significant problems" (p. 426). Because GIC embodies so much of the research regarding how mathematics should be taught to improve achievement in mathematics and so many of the instructional design features of Keller's ARCS model, it merits further study as a pedagogical model for STEM instruction and learning. Research on mathematics teaching and learning indicates important topics to address, grade-level sequences to follow, and broad instructional strategies to employ. However, more empirical studies are needed to determine the effect of innovative instructional approaches inside schools that show promise toward developing mathematical proficiency by altering the traditional interactions between teachers, students, and content.

Experiential Mathematics Instruction

There is a gap between the research on mathematics education and secondary classroom instructional practices. As described previously, the NRC (2001) and the NMAP (2008) compiled rich descriptions of effective approaches and broad strategies for improving mathematical achievement based on analyses of thousands of studies related to mathematics teaching and learning. However, mathematics instruction in U.S. classrooms today still largely follows a recitation model that rejects the learning progression theory of Bruner and the motivational design theory of Keller (Bruner, 1977; Grouws & Smith, 2000; Keller 1987a; Stigler et al., 1999). The 2008 NCTM Research Agenda Conference was an attempt to bridge the gap between research on mathematics education and classroom practice by building a community among mathematics researchers and practitioners (Arbaugh et al., 2009). Conference participants identified the needs of practitioners and developed twenty-five questions to guide research in mathematics teaching and learning (Arbaugh et al., 2009). One of the research-guiding questions was, "In what ways do different curricular approaches support or impede students' development of mathematical proficiency?" (Arbaugh et al., 2009, p. 13). Following are examples of innovative, non-traditional, experiential curricular approaches to mathematics teaching and learning that may impact the development of mathematical proficiency. The authors acknowledge the sparse number of scholarly, peer-reviewed, empirical studies to support these programs and present this literature gap as evidence that these programs merit further study. These programs were chosen because few examples exist in the literature of non-traditional, experiential learning-based mathematics curricula designed for the secondary level. In addition, these examples address many of the key recommendations for improving achievement in mathematics and motivation to learn mathematics that were uncovered during the literature review such as: (a) integrating various instructional approaches that simultaneously involve multiple strands of mathematical proficiency (NRC, 2001); (b) utilizing problem solving as a means for learning mathematics and not just a skill to be learned (Sherman et al., 2013); (c) incorporating specific instructional design components to enhance motivation (Keller, 1987a); and (d) providing authentic, relevant contexts for learning that emphasize human interactions and opportunities to build confidence and reduce anxiety surrounding learning mathematics (Beilock, Gunderson, Ramirez, & Levine, 2010; Eccles & Wang, 2016; Ganley & Lubienski, 2016).

Geometry In Construction (GIC). Geometry In Construction is an experiential learning geometry course in which students learn and apply principles of geometry, industrial technologies, and career technical education through the design and construction of a small-scale house that is donated to a local charity serving the needs of homeless people in the community (Contextual Learning Concepts, n.d.a). Contextual

Learning Concepts, LLC. developed the curriculum and aligned it to Common Core State Standards (Contextual Learning Concepts, n.d.a). Students enrolled in GIC learn all the standards addressed in traditional geometry courses within the context of construction projects and engage in both classroom and construction-related work on a daily basis (Rentsch, 2018). Geometry In Construction addresses all four components of Keller's (1987a) ARCS theory of motivational design. The instructional approaches and activities strive to capture student attention through group work and hands-on activities while providing relevance through the design and construction of real houses for community members (Taketa, 2017). Geometry In Construction also offers daily opportunities for students to build confidence and reduce the anxiety associated with learning mathematics as they work in small teams to solve problems associated with designing and building the house (Taketa, 2017). The final project, a small house that is donated to a charity serving the needs of homeless members of the community, may provide students with a sense of satisfaction that their efforts to learn geometry were worthwhile and connected to a humanitarian effort (Taketa, 2017). The instructional design of GIC addresses attention, relevance, confidence, and satisfaction; all four components Keller's ARCS theory describes as necessary to initiate and sustain motivation to learn within a specific context (Keller, 1987a).

Algebra I in Manufacturing Processes, Entrepreneurship, and Design (AMPED).

AMPED is an experiential learning, career technical education-based course in which students apply mathematical, engineering, and business management concepts to explore and solve authentic problems (Contextual Learning Concepts, n.d.b). Contextual

Learning Concepts, LLC. developed the curriculum to address the same standards as traditional algebra I courses and aligned it to the Common Core State Standards (Contextual Learning Concepts, n.d.b). The AMPED curriculum requires students to “develop a viable, self-funded business running an advanced fabrication lab customizing textile products and manufacturing other items comprised of wood, metal, and/or plastic… proceeds generated from the business aspect of the program self-fund the venture and provide philanthropic opportunities for students through community service or monetary gifts to local charities” (Contextual Learning Concepts, n.d.b, para. 2). AMPED also addresses all four components of Keller’s (1987a) ARCS theory of motivational design. The program attempts to gain student attention with group work and hands-on activities while providing relevance through the operation of a real business and applications of mathematics and engineering to create real merchandise (Contextual Learning Concepts, n.d.b). AMPED provides opportunities for students to build confidence and satisfaction as they develop authentic workplace skills bringing products to market, generating revenue to keep the company going, and engaging in community outreach and philanthropy (Contextual Learning Concepts, n.d.b). Similar to GIC, AMPED has the potential to impact motivation differently than traditional instruction and merits further investigation.

Drama-Based Geometry. Drama-Based Geometry is an experiential learning course based upon constructivist principles whereby students apply drama education techniques to construct their knowledge and understanding of geometry principles through individual and small group performances in front of classroom audiences (Ubuz & Duatepe-Paksu, 2016). Students in Drama-Based Geometry classes draw upon their own real or imagined experiences to integrate mental and physical activities that publicly demonstrate conceptual understanding of geometric principles (Ubuz & Duatepe-Paksu, 2016). Instruction in Drama-Based Geometry differs greatly from traditional geometry instruction especially in the emphasis on high levels of interpersonal relationships and experiences. In a meta-analysis of 47 quasi-experimental studies, Lee, Patall, Cawthon, and Steingut (2015) concluded drama-based pedagogy has a significant impact on achievement in educational settings with the most impact noted in language arts and science curricula. However, differences in experimental designs, composition of samples, and the criteria used to determine improvements in achievement among the 47 empirical studies do not offer strong support to their conclusion and indicates the need for further investigation of this non-traditional curricular approach. The application of Keller's (1987a) ARCS theory of motivational design could provide a uniform measure of the impact Drama-Based Geometry has on attention, relevance, confidence, and satisfaction as they relate to motivation to learn within this specific context.

Several studies agree the main goal of mathematics instruction is to develop mathematical proficiency and use it to solve problems (NMAP, 2008; NRC, 2001; Sherman et al., 2013). However, there are gaps in the literature regarding specific activities and interactions between teachers, students, and instructional materials within classrooms that might lead to mathematics proficiency. Furthermore, there is also a gap between what is known from research and what is practiced in mathematics classrooms (Arbaugh et al., 2009). GIC, AMPED, and Drama-Based Geometry are innovative curricular approaches that potentially bridge the gap between research and practice because they incorporate many of the recommendations current researchers say are important for developing mathematical proficiency. These mathematics programs warrant further study to determine the effect they have on achievement, motivation, and the development of mathematical proficiency in addition to their potential to serve as pedagogical models that successfully prepare students to pursue STEM careers.

Mathematics Achievement and the Underrepresentation of Women in STEM

Women represent approximately 50% of the college-educated U.S. workforce, but less than 30% of STEM workers are women (National Science Board, 2016). Because mathematics is a core component of STEM disciplines and careers, achievement in mathematics may impact the decision of women to pursue STEM. Researchers have investigated differences in the achievement of males and females in mathematics throughout the past 70 years (Benbow & Stanley, 1980; Wang et al., 2013). Overall, achievement differences between males and females are small, but males tend to outperform females more significantly within subgroups of higher performing students and on more advanced mathematical concepts such as problem solving and spatial reasoning (Ganley & Lubienski, 2016). Hutchison, Lyons, and Ansari (2018) studied the basic numeric skills of 1,391 boys and girls age 6-13 and found, "a male advantage in foundational numerical skills is the exception rather than the rule" (p. 66). Additionally, the mathematics average scores in the U.S. for fourth and eighth grade students on the 2017 NAEP and the 2015 TIMSS are nearly identical for males and females (U.S. Department of Education, Institute of Education Sciences, 2019). However, standardized test data continue to confirm a persistent achievement gap, with boys outperforming girls, that develops in middle school and widens in secondary school (NRC, 2013; U.S. Department of Education, Institute of Education Sciences, 2019). Data from the 2015

PISA mathematics literacy test, which tests students at age 15, and the 2015 TIMSS advanced mathematics test, which tests students in their final year of secondary school, confirm a widening gender gap in older students with males performing significantly better than females ($p<.05$) (NCES, 2019b). The literature on gender-based achievement in mathematics suggests several interrelated factors may contribute to the differences in scores.

Current research increasingly points to a variety of environmental and contextual concerns that may indirectly affect achievement in mathematics by directly affecting the interest and motivation of females to learn mathematics in middle and secondary school (Andreescu, Gallian, Kane, & Mertz, 2008; Ganley & Lubienski, 2016; National Academies of Sciences, Engineering, and Medicine, 2019; NRC, 2013; Wang et al., 2013). In a longitudinal study of 7,040 students from third through eighth grade, Ganley and Lubienski (2016) discovered that females lag behind males in confidence, interest, and achievement associated with mathematics, but the gender-based gap in confidence toward mathematics was larger than the gender-based gap in mathematical interest and achievement. Because Keller's (1987a) ARCS theory of motivation links confidence to a situational motivation to learn, the work of Ganley and Lubienski (2016) suggests the need for earlier interventions focused on building confidence in females to learn mathematics and more research to determine the effect it has on achievement in mathematics. Eccles and Wang (2016) compared occupational and lifestyle values to aptitude in twelfth grade students and found that occupational and lifestyle values were better predictors of STEM career decisions than aptitude. Further, women pursuing STEM careers are more likely to enter fields involving human interaction such as health, biological, and medical sciences than they are to enter fields focused on objects such as mathematics, physics, engineering, and computer sciences (Eccles & Wang, 2016). These results have important implications for mathematics pedagogy. If mathematics instruction emphasized human interactions and better illustrated the positive impact mathematics has on humans and society, perhaps females would perceive mathematics as more aligned with their values and be more interested in pursuing mathematics learning and STEM careers. This line of reasoning is supported with the theoretical framework provided by Keller's ARCS theory of motivation (Keller, 1987a). Keller's ARCS theory describes instructional design features that motivate learners through activities they perceive as personally relevant and satisfying (Keller, 1987b). In a study of participants in the International Mathematical Olympiad, Andreescu et al. (2008) discovered "some East European and Asian countries produce girls with profound ability in mathematical problem solving; most other countries, including the United States, do not" (p. 1258). This finding supports the argument that males and females have similar capacities for learning mathematics. The fact that females from certain cultures and geographical regions exhibit advanced mathematical ability suggests achievement in mathematics may be related more to environmental factors than intrinsic, biological, gender-based differences. The study of GIC also offers promise that the U.S. could tap into a wealth of talent by finding and addressing the environmental factors that have kept females and minorities severely underrepresented in mathematics and STEM fields. Another significant environmental impact on the achievement of females in mathematics is gender-based stereotypes. In a study of 117 first and second grade students, Beilock et al. (2010) found that females, but not males, are more likely to accept the stereotype, "boys are good at math, girls are good at reading" (p. 1860) and consequently demonstrate lower achievement in mathematics toward the end of one year of instruction from a female teacher exhibiting math anxiety. Again, this shows how an environmental and contextual factor, being taught by a female with math anxiety, can influence beliefs and attitudes about oneself and impede the achievement of females in mathematics. This finding also highlights the need for innovative mathematics instruction that builds confidence, reduces anxiety, and aligns with the personal values of females. While most gender-based achievement differences in mathematics appear to develop as the result of social, cultural, environmental, and contextual pressures on females, there is one contextual area in which males do seem to have an innate advantage.

There is some evidence that males outperform females in the specific mathematical context of spatial reasoning (Tzuriel & Egozi, 2010). Spatial reasoning, the ability to envision, orient, and manipulate objects in three-dimensional space, has been used as a predictor of achievement in mathematics and future success in STEM for a long time (Lowrie & Jorgensen, 2018; Wai, Lubinski, & Benbow, 2009). In a study of 116 children in first grade, Tzuriel and Egozi (2010) showed gender differences in spatial reasoning ability can be reduced through specific instructional strategies. Other studies confirm improvements in spatial reasoning as a result of instructional interventions (Lowrie, Logan, & Ramful, 2017; Uttal et al., 2013). These studies are pivotal in the discussion about gender-based achievement differences in mathematics because spatial reasoning ability is such a strong predictor of achievement in mathematics and success in STEM. If spatial reasoning can be improved through instructional interventions, these interventions can be leveraged to improve the achievement of females in mathematics and thus increase the number of females prepared to succeed in learning and pursuing careers in STEM. Geometry seems a logical course for interventions that improve spatial reasoning because learning geometry requires students to manipulate shapes, objects, and angles in three dimensional space.

The current literature does not agree on whether gender-based achievement differences in mathematics exist. Some studies suggest the gender differences are small, and males and females exhibit similar performance in mathematics (Ganley & Lubienski, 2016; Lindberg, Hyde, Petersen, & Linn, 2010). In contrast, Cimpian et al. (2016) indicated a gender gap persists but is somewhat masked early on in school by the stronger learning efforts of females to do well and get good grades. The literature does suggest that males tend to outperform females in secondary school and college, in subgroups of high performing students, and in advanced concepts such as problem solving and spatial reasoning (Ganley & Lubienski, 2016; NRC, 2013; U.S. Department of Education, Institute of Education Sciences, 2019). The causes of gender-based achievement gaps in mathematics seem to lie more within the intersection of social, cultural, contextual, and environmental influences on females rather than innate cognitive differences among genders. These environmental influences result in many females having low confidence and high anxiety associated with learning mathematics (Beilock et al., 2010; Eccles & Wang, 2016; Ganley & Lubienski, 2016). Consequently, many females are not motivated to learn mathematics or pursue careers in mathematics and STEM (Eccles & Wang, 2016; PCAST, 2012). In order to improve achievement in mathematics among women in secondary school and college, and thus increase the number of women prepared to succeed in STEM careers, it is necessary to begin interventions at the elementary and middle school level (Arbaugh et al., 2009; Eccles & Wang, 2016; Lowrie & Jorgensen, 2018). One form of intervention could be innovative mathematics curricula implemented by highly qualified teachers, that aligns with the values of females, builds confidence, and reduces the anxiety associated with learning mathematics for some females. In addition, all mathematics curricula and instructional approaches need to directly address the environmental and contextual influences that impede the achievement of females in mathematics. Mathematics instruction emphasizing interactions with people and the benefits mathematics provides to humans and society may align better with the personal and occupational values of females and motivate more females to pursue careers in STEM (Eccles & Wang, 2016).

Summary

Historically, mathematics instruction oscillates between teacher-directed and student-centered approaches every decade or two (Fey & Graeber, 2003; Klein, 2003). However, large meta-analyses of research on mathematics teaching and learning highlight the merits of both and support integrated approaches that balance the time spent developing conceptual understanding, problem solving, and procedural skills (NMAP, 2008; NRC, 2001). Unfortunately, standardized test data show flat growth in the achievement of U.S. students in mathematics over the last 30 years and indicates a gap between the knowledge obtained from research on mathematics teaching and learning and the practices implemented in classrooms (Arbaugh et al., 2009; NCES, 2019b; U.S. Department of Education, Institute of Education Sciences, 2019). Several studies agree the development of mathematics proficiency and problem solving ability are the ultimate goals of mathematics teaching and learning and outline broad strategies for achieving these goals (NMAP, 2008; NRC, 2001; Sherman et al., 2013). However, the literature is sparse regarding specific strategies and interactions between teachers, students, and instructional materials inside the classroom that lead to mathematical proficiency (Arbaugh et al., 2013; NRC, 2001;). The NRC (2001) concluded, "mathematical proficiency for all demands that fundamental changes be made concurrently in curriculum, instructional materials, assessments, classroom practice, teacher preparation, and professional development" (p. 10). Geometry in Construction, an experiential geometry course, embodies those changes through the implementation of innovative teaching and learning strategies grounded by Keller's (1987a) ARCS theory of motivation. The GIC curriculum merits further investigation to measure the effect it has on achievement in geometry and motivation to learn geometry. Finding pedagogical models that increase achievement in all branches of mathematics is important for enhancing interest in STEM and successfully preparing students to pursue STEM careers (PCAST, 2012).

CHAPTER 3

METHODOLOGY

Chapter three outlines and explains the methods used to test the hypotheses and answer the research questions. The methodology focused on investigating the effects of an experiential learning course called Geometry In Construction (GIC) on secondary student achievement in geometry and motivation to learn geometry. The overall objective is to explore instructional strategies that might increase the number of secondary and college students pursuing STEM careers and better prepare them for success in STEM. Discussion of the research design, research questions, sample, instrumentation, and methods of data collection and analysis follows.

Research Design

A quantitative research approach was used to analyze data collected near the completion of the second semester of an experiential geometry course called Geometry In Construction and a traditionally taught geometry course in order to measure the effects of Geometry In Construction on secondary student achievement and motivation to learn geometry. Quantitative research involves following a post-positivist worldview by which phenomena are observed and measured by collecting numerical data (Creswell, 2014; Grix, 2010). Creswell (2014) goes on to explain that quantitative research often focuses on identifying variables and examining the relationships among them. Quantitative researchers identify an independent variable, which may be a treatment or intervention that can be administered to a sample, and then quantify the association or effect it has with or on other variables known as dependent variables. The influence of additional variables must be strictly addressed by a research design that controls these additional influences and specifically explains the impact of other factors that may moderate results (Creswell, 2014).

A quasi-experimental design was used to compare data from an experimental and a control group. A quasi-experimental design was chosen because the participants in each group could not be randomly assigned. When participants are not randomly assigned to the treatment and control groups, it is possible that the two groups could be dissimilar. Therefore, a non-equivalent, pretest and posttest control-group design was used "in which both groups take a pretest and posttest, but only the experimental group receives the treatment" (Creswell, 2014, p. 172). This research design is similar to a true experimental pretest and posttest control group design except that the participants were not randomly assigned to the treatment and control groups. The experimental group consisted of students enrolled in a GIC course. The control group consisted of students enrolled in a traditional geometry course. The goal of the quantitative data was: (a) to determine if experiencing the GIC curriculum (independent variable) has an effect on achievement in geometry (dependent variable) and motivation to learn geometry (dependent variable); (b) to determine if experiencing the GIC curriculum (independent variable) affects the achievement (dependent variable) and motivation (dependent variable) in geometry of males and females differently (gender is an independent variable). Two measures of quantitative data were collected upon the conclusion of the experimental treatment. Scores on a Missouri Geometry End of Course (EOC) Practice Exam were used to compare the achievement of students experiencing a GIC curriculum to students experiencing a traditional geometry curriculum. John Keller's (2010) Course

Interest Survey (CIS) was used to compare motivation to learn geometry of students experiencing a GIC curriculum to students experiencing a traditional geometry curriculum. All participants previously completed an algebra I course and took the Missouri Algebra I EOC Exam; therefore, scores on the algebra I EOC exam served as the pretest (covariate) to compare the experimental and control groups prior to implementing the treatment.

A visual model of the quasi-experimental, nonequivalent pretest and posttest control-group experimental design is shown in Figure 7 (Creswell, 2014). The model employs the notation system of Campbell and Stanley (1963) where “X” represents a treatment for which effects were measured and “O” represents a measured event. The vertical alignment of both measured events, “O”, indicates the pretest and posttest were administered to the experimental group and the control group at the same time. The horizontal line between the experimental and control group signifies the groups were not randomly assigned (Campbell & Stanley, 1963).

Group A O–––––X–––––O

Group B O–––––––––––O

time ⟶

Figure 7. Visual model of nonequivalent pretest and posttest control-group design. O represents measured event, X represents treatment. Adapted from *Handbook of research on teaching*, by D.T. Campbell and J.C. Stanley, Copyright 1963 by Rand McNally.

Internal Threats to Validity

Sound research design minimizes internal threats to validity that challenge whether the outcomes of experiments were related to the intervention or other factors (Creswell, 2014). Internal threats of maturation, selection, and mortality involve the participants (Creswell, 2014). All participants were in ninth or tenth grade which minimized the effect of maturation as all participants were of similar age and were assumed to mature similarly throughout the academic year as they experienced a geometry curriculum. A large, heterogeneous sample size (N=181) consisting of ninth and tenth grade males and females and the use of pretest algebra I EOC exam scores as a covariate minimized the threat of selection because students with characteristics that could skew outcomes were distributed throughout the treatment and control groups and controlled for with statistical treatment. Mortality was also addressed with a large sample size that would minimize the effect of students who withdrew during the study. Diffusion of treatment, resentful demoralization, and compensatory rivalry are additional threats to the internal validity of experimental designs that have treatment and control groups (Creswell, 2014). These threats occur when participants of one group communicate with another group and influence the outcomes of experiments (Creswell, 2014). Internal threats related to the experimental treatment were minimized by selecting participants near the completion of the academic school year, thereby restricting the amount of time participants had to communicate with members of the other group. In addition, participants were not informed that geometry curricula were being compared, nor that GIC was considered the treatment and traditional geometry was considered the control. Furthermore, until data were collected and analyzed, there was no indication that either curriculum was beneficial or harmful, therefore the control group did not have reason to resent or consider the treatment group to be a rival. Instrumentation can also be an internal threat to validity if the instrument changes between the pretest and posttest (Creswell, 2014). The Missouri Algebra I EOC Exam served as the baseline measure of achievement in mathematics, and a Missouri Geometry EOC Practice Exam served as the posttest measure of achievement in geometry. The instrumentation threat to internal validity was addressed by showing (in the instrumentation sub-section) that these tests demonstrate convergent validity, high correlation, and scores on the algebra I EOC exam accurately predict scores on the geometry EOC exam (Egan, 2012; Questar, 2017).

External Threats to Validity

Creswell (2014) explained "external validity threats arise when experimenters draw incorrect inferences from the sample data to other persons, settings, and situations" (p. 176). The main threats to external validity involve the selection of participants and setting. It is important to note the participants were predominantly White students, age 14-16, who passed an algebra I course. The setting is a large public high school in a large suburban district in Missouri. The GIC courses and the traditional geometry courses at the research site are taught by veteran teachers each having more than 10 years of experience teaching geometry. In order to minimize threats to external validity, generalizations about the results will not be made beyond the specific population studied.

Research Questions

1. What effect does experiencing the Geometry In Construction curriculum have on the achievement in geometry of secondary students compared to experiencing a traditional geometry curriculum as measured by the Missouri Geometry End of Course Exam?

2. Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?
3. What effect does experiencing the Geometry In Construction curriculum have on the motivation of secondary students to learn geometry compared to experiencing a traditional geometry curriculum as measured by Keller's (2010) Course Interest Survey?
4. Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?

Hypotheses

H_0 1: There is no significant difference between the achievement in geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by Missouri Geometry End of Course Exam scores.

H_0 2: Experiencing the Geometry In Construction curriculum does not affect the achievement in geometry of secondary males and females differently as measured by Missouri Geometry End of Course Exam scores.

H_0 3: There is no significant difference between the motivation to learn geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by scores on Keller's (2010) Course Interest Survey.

H_0 4: Experiencing the Geometry In Construction curriculum does not affect the motivation of secondary males and females to learn geometry differently as measured by scores on Keller's (2010) Course Interest Survey.

Population and Sample

The population of interest was all ninth and tenth grade students enrolled in GIC and traditional geometry courses at a large, suburban, Midwestern, public high school during the 2018-19 academic school year. The overall demographics of the high school in 2018 included an enrollment of approximately 1300 students consisting of 80% White, 10% African American, 3% Asian, 3% Hispanic, and 4% other or multiple race (MO DESE, 2019b). In 2018, 17.4% of the school population received free or reduced lunch, and the school had an overall graduation rate of 96% (MO DESE, 2019b).

The high school offers two sections of GIC, each taught by teachers having more than 20 years of experience teaching mathematics. A convenience sample of all 58 students enrolled in both sections of GIC was selected for the treatment group to attain a sample large enough to minimize the effects of mortality and sampling error. A random sample could not be obtained because secondary students self-select their courses. The treatment sample consisted of 35 males, 23 females, 24 ninth grade students, and 34 tenth grade students. The high school offers five sections of traditional geometry, taught by two different teachers each having more than 10 years of experience teaching mathematics. A convenience sample of all 123 students enrolled in traditional geometry was selected for the control group. The control group consisted of 45 males, 78 females, 20 ninth grade students, and 103 tenth grade students. The total sample (N=181) consisted exclusively of ninth and tenth grade students who successfully completed an algebra I course prior to enrollment in geometry.

Participants were recruited in person by the researchers during site visits to the GIC and traditional geometry classrooms. During the site visits, it was explained that the study is an evaluation of how their geometry course impacts their achievement and motivation to learn geometry. Student assent and parental consent forms were distributed during the site visit and students were given two weeks to return the signed forms if they chose to participate.

Students in the treatment group experienced a GIC curriculum that integrates geometry with principles of industrial and career technical education (Contextual Learning Concepts, n.d.a). The curriculum was originally developed by Contextual Learning Concepts, LLC. and aligned to the Common Core State Standards for Mathematics (Contextual Learning Concepts, n.d.a; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). The curriculum implemented at the research site aligns to the Missouri Learning Standards which are similar to the Common Core State Standards for Mathematics as illustrated in the comparison shown in Appendix A. Students experiencing the GIC curriculum learn geometry in the context of building a small house that they donate to a local charity serving the needs of homeless community members. Students also learn and develop workplace skills for the 21st century by working in small teams of three to four students. The GIC course meets for two 54-minute class periods each day, or 540 minutes per week for 36 weeks, for an overall total of 324 hours. Generally, students spend one of the class periods in a classroom planning and designing components of the construction project. Students spend the other daily class period outside at the construction site building the house. During both periods, students work in small groups learning and applying the same principles of geometry their peers learn in a traditional classroom setting. The experienced mathematics teachers who facilitate the GIC classes assign homework problems almost daily, and students must complete each assignment before they can work on the house that day. Students in the control group experienced a traditional geometry curriculum aligned to the Missouri Learning Standards. The traditional geometry courses meet for one 54-minute class each day, or 270 minutes per week for 36 weeks, for an overall total of 162 hours. Students enrolled in traditional geometry experience recitation-type daily instruction generally consisting of a brief homework check or quiz, lecture presentation of information, guided in-class practice problems often completed in small groups, and additional practice problems assigned for homework and due the following day. Instruction in both GIC and the traditional geometry course incorporates the same curricular standards for geometry, and students in both classes take identical unit assessments and semester final exams. Figure 8 compares the teaching methodologies of both classes. Students experiencing the GIC curriculum do not receive more instructional time, tutoring, or homework than students experiencing the traditional geometry curriculum. The additional time embedded into the GIC curriculum is spent incorporating the experiential aspect of the class (designing, building, and constructing). Therefore, it is not believed that time spent teaching and learning geometry is influencing the outcome as much as the fact that the GIC curriculum includes an experiential component and the traditional geometry curriculum does not.

Curriculum Component	**Geometry In Construction**	**Traditional geometry**
Curriculum type	Experiential	Traditional (teacher-directed)
Curriculum alignment	Common Core Standards, Missouri Learning Standards	Common Core Standards, Missouri Learning Standards
Course length	Two semesters, 36 weeks total	Two semesters, 36 weeks total
Time in class	108 min. per day 324 hrs. total per year	54 min. per day 162 hrs. total per year
Teaching methodologies	Small-group classroom, shop, and construction site lecture as needed, practice, daily homework plan, design, construct weekly quizzes, unit tests every 3-4 weeks, final exam each semester	Individual classroom daily lecture, practice, daily homework notes, guided practice, individual practice weekly quizzes, unit tests every 3-4 weeks, final exam each semester

Figure 8. Comparison of key curricular components of Geometry In Construction and traditional geometry.

Instrumentation

The following section describes the instruments that were used to measure the effects of geometry curriculum and gender on the two dependent variables of achievement in geometry and motivation to learn geometry.

Achievement in Geometry

A pretest/posttest design was used to measure achievement in geometry after experiencing a full academic year of either GIC or traditional geometry curriculum. Scores from the 2018 Missouri Algebra I EOC Exam were used to establish and compare the baseline achievement in mathematics between the treatment and control groups. A Missouri Geometry EOC Practice Exam (see Appendix B) was used as the posttest instrument to compare achievement in geometry after experiencing a geometry curriculum. The algebra I EOC exam is a Missouri state requirement for high school graduation (MO DESE, 2019a). At the research site, the algebra I EOC exam was administered three weeks prior to the end of algebra I courses, which for all participants, was during eighth or ninth grade. The algebra I EOC exam was administered online and scored by Questar Assessment, Inc. through a contract with the Missouri Department of Elementary and Secondary Education (MO DESE, 2019a). The geometry EOC exam was administered during the final week of geometry courses, but since it is not a Missouri state requirement for high school graduation, it was administered as a paper and pencil exam and scored by the classroom geometry teachers. Questar Assessment, Inc. develops EOC exam scale scores using a proprietary formula based on correct responses and their point values to indicate four levels of achievement: below basic, basic, proficient, and advanced (Questar, 2018). The scale scores established for the 2018 Missouri Algebra I EOC Exam are shown in Table 1 (Questar, 2018). A raw-to-scale-score converter,

Table 1

Scale Scores for the 2018 Missouri End of Course Exam in Algebra I

Achievement Level	**Scale Score**
Advanced	409 and above
Proficient	400 - 408
Basic	389 - 399
Below Basic	325 - 388

Note. Adapted from *End of Course Assessments: Technical Report.* Copyright 2018 by Questar Assessment, Inc.

published by Questar Assessment, Inc. (2018) was used to convert the algebra I scale scores to raw scores (points correct). Raw scores were then converted to percentages of the total points possible for both the algebra I and geometry EOC exam scores to provide equivalent comparisons of the baseline and posttest data.

In a study examining the relationships between Missouri EOC exam scores from 2009, 2010, and 2011, Egan (2012) established the reliability of algebra I and geometry EOC exam scores using Cronbach's (1951) coefficient alpha. Cronbach's alpha provides a measure of internal consistency by indicating the extent to which a group of questions measure the same construct (Laerd, 2018a). Values for Cronbach's alpha can range from zero to one with values closer to one indicating higher consistency and values greater than 0.8 generally considered acceptable (Egan, 2012). The Cronbach's alpha values were .86, .86, and .87 for the 2009, 2010, and 2011 algebra I EOC exam administrations, respectively (Egan, 2012). The geometry EOC exam was not administered in 2009, but Cronbach's alpha values were .87 and .82 for the 2010 and 2011 geometry EOC exam administrations, respectively (Egan, 2012). A different analysis of 2017 EOC exam data indicate Cronbach's alpha values of .88 and .85 for the algebra I and geometry EOC exams, respectively (Questar, 2017). Creswell (2014) describes reliability as "whether or not response scores to items on the instrument are stable over time and whether there was consistency in test administration and scoring of the instrument" (p. 160). Consistently high year-to-year Cronbach alpha values for the algebra I and geometry EOC exams demonstrate sustained internal consistency and thus provide a measure of reliability for both assessments.

Egan (2012) also found the algebra I and geometry EOC exams to be "strongly related to each other, thus demonstrating convergent validity, and moderately related to other subject area EOC exam scores suggesting divergent validity" (p. 20). Convergent validity relates to "the extent to which theoretically related constructs are empirically related while divergent validity relates to the extent to which theoretically unrelated constructs are not empirically related" (Egan, 2012, p. 2). Egan (2012) used a Pearson product moment correlation coefficient to measure divergent validity. The Pearson correlation coefficient, r, represents the extent to which two variables are related to each other and ranges from -1 to +1 with negative values indicating an inverse relationship between the variable, positive values indicating a direct relationship between the variables, and zero indicating no relationship between the variables (Laerd, 2018d). The strength of relationship between two variables is generally classified as strong, moderate, or weak. Pearson correlation coefficient, r, values greater than .70 are considered strong, between .30 and .69 are considered moderate, and below .29 are considered weak (Egan, 2012). In an analysis of paired data, comparing the algebra I EOC exam scores of students to their geometry EOC exam scores in a later year, Egan (2012) found a strong correlation ($r>.70$). The Pearson correlation coefficient associated with the comparison of 2009 algebra I EOC exam scores to 2010 geometry EOC exam scores was .72 (Egan, 2012). The same Pearson correlation of .72 was found when comparing 2010 algebra I EOC exam scores to 2011 geometry EOC exam scores (Egan, 2012). In separate analyses performed by Questar Assessment, Inc., Pearson correlation coefficients of .82 and .74 were found when comparing algebra I and geometry EOC scores in 2017 and 2018, respectively (Questar, 2017, 2018). Pearson coefficients of .72, .72, .82, and .74 suggest a strong correlation between algebra I and geometry EOC exam scores and demonstrate convergent validity. Comparisons of 2017 EOC exam scores between algebra I and American history ($r = .47$) and between geometry and English I ($r = .52$)

reveal Pearson correlation coefficients in the moderate range and demonstrate divergent validity (Questar, 2017). The convergent and divergent validity described by Egan (2012) also establishes construct validity since they are both subtypes of construct validity.

Additional evidence of construct validity for the algebra I and geometry EOC exams is provided by examining the test and item development. Creswell (2014) describes construct validity as the extent to which an instrument measures what it purports to measure. According to the Missouri Department of Elementary and Secondary Education (MO DESE), "End of course assessments measure how well students acquire the skills and knowledge described in Missouri's Learning Standards. The assessments yield information on academic achievement at the student, class, school, district and state levels to gauge the overall quality of education throughout Missouri" (MO DESE, 2019a, para. 3). In a technical report presented to the Missouri DESE, Questar Assessment (2017) analyzed the development of Missouri EOC tests and test items. Questar (2017) verified "adequate representation of the Missouri Learning Standards is ensured in every EOC exam through the use of a test blueprint and a documented test construction process" (p. 135). Questar (2017) examined the Missouri EOC test construction process to ensure "test items covered an array of contexts and cultures, there were sufficient test items distributed across content and varying difficulty levels, test writers were trained, test items were reviewed by content experts and properly aligned to standards and grade levels, and teachers from diverse ethnic and geographical backgrounds reviewed item accessibility" (p. 135). In addition, the Missouri DESE commissioned two external alignment studies to ensure the assessments represented the

Missouri Learning Standards and measured student knowledge at the depth of knowledge described in the standards. The first study concluded that the 2009 EOC test forms were "fully aligned for most criteria", and the second study concluded that all test forms were "partially or fully aligned for all criteria" (Questar, 2017, p. 136). The GIC and traditional geometry curricula implemented at the study site are aligned to the Missouri Learning Standards for geometry; therefore, the Missouri Geometry EOC Exam is a valid measure of achievement in those courses.

Motivation to Learn Geometry

A survey instrument was used to measure motivation to learn geometry after experiencing a full academic year of either the GIC or traditional geometry curriculum. The survey instrument used to measure motivation to learn geometry is John Keller's Course Interest Survey (CIS), shown in Appendix C (Keller, 2010). The CIS is based on Keller's (1987a) ARCS theory of motivation which relates the four components of attention, relevance, confidence, and satisfaction to a situational motivation to learn. Keller (2010) designed the CIS to measure motivation of students in response to a specific course or learning condition; therefore, it is not intended to characterize overall motivation for learning, but rather motivation within a specific, situational context. The survey was designed for use with secondary students, college students, and adults (Keller, 2010). The CIS consists of 34 questions divided into four subsections allowing separate, subscale scores and means to be calculated for attention, relevance, confidence, and satisfaction. Participant responses to each question are recorded on a five-point Likert scale indicating the degree to which participants agree or disagree with the statement (1 = not true, 2 = slightly true, 3 = moderately true, 4 = mostly true, 5 = very true). Nine questions have a reverse-loaded scale. The CIS was administered to participants online, under the direct supervision of their geometry teacher, during the last week of their geometry course. Keller grants permission to use the survey and adapt it to specific situations by changing phrasing such as "this course" to "this geometry course" (Keller, 2010). Other than phrasing to customize the CIS to specific courses, the items were not modified as each one is linked to specific components of motivation (Keller, 2010).

Keller (2010) described the efforts to establish reliability and validity for the CIS instrument. The initial development of the survey included assembling a large pool of questions that were reviewed by ten graduate students knowledgeable about Keller's ARCS model and the current literature on motivation (Keller, 2010). Many questions were revised, edited, and deleted due to ambiguity or misalignment before being tested by a different set of graduate students (Keller, 2010). The questions were analyzed for reliable phrasing and construct alignment by having the graduate students pretend to answer the questions as if they were motivated by a course and then again as if they were unmotivated by a course (Keller, 2010). Based on the results of the second analysis, questions were revised, edited, or deleted once again before a final round of testing on the remaining 34 items to ensure they could accurately and reliably discriminate the four components of Keller's ARCS theory of motivation (Keller, 2010).

Additional studies were conducted to establish the reliability of the CIS by using Cronbach's alpha as a measure of internal consistency. The survey was initially administered to a test group of 45 undergraduate students and then to a test group of 65 undergraduate students (Keller, 2010). Data from these tests indicated a need to further revise the instrument (Keller, 2010). In a large scale pilot test, Keller (2010) "administered the revised CIS to 200 undergraduate and graduate students in the School of Education at the University of Georgia at Valdosta and also collected student course grades and grade point averages" (p. 5). Analysis of the pilot test data resulted in Cronbach's alpha values of .84, .84, .81, and .88, for the motivation subscales of attention, relevance, confidence, and satisfaction, respectively, and .95 for the overall scale as shown in Table 2 (Keller, 2010). The values for Cronbach's alpha can range from zero to one with values closer to one indicating higher consistency and values greater than 0.8 generally considered acceptable (Egan, 2012).

Table 2

Cronbach's Alpha Values Measuring Internal Consistency of CIS Pilot Test

Scale	**Reliability Estimate (Cronbach's alpha)**
Attention	.84
Relevance	.84
Confidence	.81
Satisfaction	.88
Total Scale	.95

Note. Adapted from Motivational design for learning and performance: The ARCS model approach, by J. M. Keller. Copyright 2010 by Springer Publishing.

Data from the pilot test also enabled Keller to establish construct validity by comparing CIS scale and subscale scores to student course grade and grade point average (GPA) as shown in Table 3. Keller (2010) calculated Spearman's rank-order correlation coefficients and found statistically significant correlations between CIS scores and student course grades ($p<.05$). In addition, Keller (2010) found there was not a statistically significant correlation between CIS scores and student GPA ($p>.05$).

Table 3

Correlation Coefficients Between CIS Scores, Course Grade, and GPA

ARCS Categories	**Course Grade**	**GPA**
Attention	.19	.01
Relevance	.43	.08
Confidence	.49	.03
Satisfaction	.49	.03
Total Scale	.47	.04

Note. Adapted from Motivational design for learning and performance: The ARCS model approach, by J. M. Keller. Copyright 2010 by Springer Publishing.

In general, correlation coefficients indicate the extent of relationship between two variables: Spearman correlation coefficient, r_s, values greater than .60 are considered strong, between .40 and .59 are considered moderate, and below .39 are considered weak (Statstutor, n.d.). Because the CIS scores were correlated with student course grade but not student GPA, the results "support the validity of the CIS as a situation-specific measure of motivation, and not as a generalized motivation measure, or construct measure for school learning" (Keller, 2010, p. 281).

Data Collection

Contextual Learning Concepts, LLC. developed an experiential geometry course called Geometry In Construction (GIC) in which students learn principles of geometry, industrial education, and career technical education in the context of building a small house that is donated to a local charity serving the needs of homeless community members (Contextual Learning Concepts, n.d.a). Interest in the impact GIC has on achievement and motivation in geometry arises from knowledge that lack of proficiency and interest in learning mathematics are often barriers to entry, persistence, and success in STEM (Lazowski & Hulleman, 2016; NCTM, 2000; NRC, 2001; PCAST, 2012; Wang et al., 2013).

Student scores on a Missouri Geometry Practice EOC Exam served as the measure of achievement in geometry. Participants completed the exam in their geometry classroom under the direct supervision of their geometry teacher during the final week of a two-semester geometry course. The geometry teachers were trained by their school district to meet all the Missouri state policies and guidelines for fairly and securely proctoring standardized state tests. The Missouri Geometry EOC Exam is not a timed test. The Missouri Algebra I EOC Exam provided a measure of baseline achievement in mathematics. Participant scores on the algebra I EOC exam were pre-existing data that were used as a covariate when analyzing geometry EOC exam scores as explained in the subsequent data analysis section. All participants took the algebra I EOC exam in their algebra I course during the academic year immediately preceding enrollment in a geometry course.

Student scores on Keller's (2010) Course Interest Survey served as the measure of motivation to learn geometry. The CIS was administered to participants in their geometry classroom under the direct supervision of their geometry teacher during week 35 of a 36 week, two-semester geometry course. The survey required approximately 30 minutes to complete and was administered online using Qualtrics. Geometry teachers were trained to administer the CIS online.

Ethics and Human Relations

The participants of the study were minors, therefore, specific guidelines involving human subject research involving children were strictly followed. A comprehensive review by the Institutional Review Board occurred prior to interacting with participants and collecting data. Participation was voluntary and void of coercion and deception. Site visits to participant classrooms were conducted to explain the research study as an analysis of the impact of geometry curricula on achievement and motivation to learn geometry. Participants were not informed about the comparison of GIC to traditional geometry curricula nor whether they would be in a treatment or control group. Student assent and parental consent forms were distributed during the site visits and collected by the geometry classroom teachers during the two weeks prior to data collection. Upon approval to conduct the study, researchers met with each geometry classroom teacher to share the general purpose of the study and train them for administering the online CIS. The geometry teachers were informed of the importance of participant confidentiality and discouraged from further discussing the study with participants or revealing the nature of the treatment and control groups as that could be a threat to the internal validity of the experimental design as discussed previously.

Neither the statistical analyses of anonymous achievement scores nor the completion of an online survey measuring student motivation to learn geometry by participants posed a significant risk to the physical, psychological, social, economic, or legal well-being of the participants. Since all of the data were collected and analyzed electronically, there was a small risk that the data could be compromised or viewed by unauthorized persons. Multiple precautionary measures were taken to protect the privacy of participants. As part of this effort, the identity of participants will not be revealed in any publication or presentation. All identifying information was removed from the achievement score data, and at no time was the identity of a particular student, their scores, or their participation revealed. The anonymous EOC exam and CIS score data will be stored securely for a period of up to three years on password protected computers that operate behind a firewall and are only accessible by the researchers and their system administrators. After three years, all EOC exam and CIS score data will be permanently deleted. The online CIS survey was taken anonymously on a secure network with no collection of identifying information; therefore, individual responses to specific survey questions were not identifiable.

Data Analysis

The data analysis involved examining the relationship between the two independent variables, geometry curriculum and gender, and the two dependent variables, achievement in geometry and motivation to learn geometry. Algebra I EOC exam scores served as the baseline measure of achievement in mathematics and geometry EOC exam scores served as the posttest measure of achievement in geometry. Participant scores on Keller's (2010) CIS served as the measure of motivation to learn geometry. An overall scale score and four subscale scores measuring participant perceptions of their attention, relevance, confidence, and satisfaction were calculated and analyzed from the CIS results. The validity and reliability of both instruments was discussed in the instrumentation section of this chapter.

The baseline achievement in mathematics of the treatment group and the control group were compared. An independent t-test comparing the mean algebra I EOC exam scores of the treatment and control groups was conducted to determine if their baseline scores were significantly different. The results of the t-test are shown in chapter 4. One-way analysis of covariance (ANCOVA) was used to analyze statistical differences in geometry EOC exam scores representing the dependent variable "achievement in geometry" in response to the independent variables of group (treatment or control) and gender (male or female) and controlling for the covariate of algebra I EOC exam scores representing baseline achievement in mathematics. One-way ANCOVA was chosen because it assesses the extent to which an independent, categorical variable (group or gender) is associated with statistically significant differences in a continuous, dependent variable (achievement in geometry as measured by geometry EOC exam scores) while controlling for a third variable called the covariate (baseline achievement in mathematics as measured by algebra I EOC exam scores) in order to remove the effect of the covariate on the relationship between the independent and dependent variables (Laerd, 2018c).

Four research models were created for ANCOVA corresponding to research questions one and two. Table 4 displays the variables, covariates, and comparison group used in each model. The data for each model were analyzed to ensure it met the underlying assumptions and criteria for ANCOVA to produce valid results. Those results are provided in Appendix D. The variables meet the criteria for proper use of ANCOVA because the dependent variable and covariate are measured on a continuous scale and the independent variables are categorical. No participants were in both the treatment and control groups. As recommended by Laerd (2018c), analyses were conducted to identify:

Table 4

ANCOVA Research Models for Analyzing Achievement in Geometry

Model	Research Question	Independent Variable	Dependent Variable	Covariate	Comparison Group
1	What effect does experiencing the Geometry In Construction curriculum have on the achievement in geometry of secondary students compared to experiencing a traditional geometry curriculum as measured by the Missouri Geometry End of Course Exam?	curriculum	achievement in geometry	Algebra I EOC Scores	control
2A	Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?	gender	achievement in geometry	Algebra I EOC Scores	GIC males compared to GIC females
2B	Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?	curriculum	achievement in geometry	Algebra I EOC Scores	GIC males compared to control males
2C	Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?	curriculum	achievement in geometry	Algebra I EOC Scores	GIC females compared to control females

“outliers, normal distribution of residuals, homogeneity of variances, linear relationship of covariate to dependent variable, homoscedasticity, and homogeneity of regression slopes” (paras. 6-16) before proceeding with ANCOVA.

Independent, two-tailed t-tests were used to analyze statistical differences in CIS overall mean scores and subscale mean scores for attention, relevance, confidence, and satisfaction representing the dependent variable “motivation to learn geometry” for the independent variables of group (treatment or control) and gender (male or female). Multiple t-tests were chosen because they can reveal which independent, categorical variables (group and gender) are associated with statistically significant differences in the means of multiple, continuous, dependent variables (motivation to learn geometry as measured by overall and subscale scores on the CIS) (Laerd, 2018b).

Four additional research models were created for t-tests corresponding to research questions three and four. Table 5 displays the variables and comparison group used in each model. The data for each model were analyzed to ensure it met the underlying assumptions and criteria for t-tests to produce valid results. Those results are provided in Appendix E. As recommended by Laerd (2018b), testing was performed to identify: “significant outliers, normality, and homogeneity of variances” (paras. 5-10) associated with dependent variable data before proceeding with t-tests. Failure of data to meet the criteria for valid use with ANCOVA and t-tests was addressed through additional statistical treatments or, where appropriate, reliance on large sample size to validate certain violations of the assumptions. Type I errors, incorrectly rejecting a null hypothesis, and type II errors, incorrectly accepting a null hypothesis, were minimized by establishing an alpha value of .05 to indicate statistically significant differences.

Table 5

t-test Research Models for Analyzing Motivation to Learn Geometry

Model	Research Question	Independent Variable	Dependent Variable(s)	Comparison Group
3	What effect does experiencing the Geometry In Construction curriculum have on the motivation of secondary students to learn geometry compared to experiencing a traditional geometry curriculum as measured by Keller's (2010) Course Interest Survey?	curriculum	motivation attention relevance confidence satisfaction	control
4A	Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?	gender	motivation attention relevance confidence satisfaction	GIC males compared to GIC females
4B	Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?	curriculum	motivation attention relevance confidence satisfaction	GIC males compared to control males
4C	Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?	curriculum	motivation attention relevance confidence satisfaction	GIC females compared to control females

Limitations

A quantitative, quasi-experimental research approach was used to investigate the effects of an experiential geometry course on achievement and motivation in geometry. A moderate size sample (N=181) was analyzed, but generalization of the results is limited to similar populations of suburban, predominantly White, Midwestern, public secondary schools. An additional factor impacting generalization of the results may be geometry teacher experience. Geometry teachers of both the treatment and control groups were experienced teachers each having more than ten years of experience teaching geometry. In addition, the two GIC teachers were in their second year of GIC curriculum implementation. Future studies are needed to replicate the study in other suburban, urban, and more culturally-diverse settings.

A convenience sample of all 123 students enrolled in traditional geometry was selected for the control group, and a convenience sample of all 58 students enrolled in GIC was selected for the treatment group. It was not possible to obtain a random sample of students because secondary students choose their own academic schedules and self-select the courses they want to take. Non-random selection of participants can limit the interpretation of findings because certain student characteristics may introduce bias. For example, it is possible that students choose to enroll in GIC because they like mathematics, are highly motivated to learn mathematical applications, want to help their community, have higher self-efficacy, or perhaps, they dislike mathematics and are trying to avoid traditional geometry which often involves more lecture and prolonged seat time for practice and problem solving (Grouws & Smith, 2000). A non-random distribution of these student characteristics can introduce bias and limit the interpretation of findings.

Additional limitations are inherent due to methodological constraints. The length of study precluded researchers from obtaining pretest measures of motivation to learn geometry. Because the geometry curricula investigated are implemented within 36-week courses, pretest measures of motivation need to be collected in August, at the beginning of the academic year, and posttest measures of motivation and achievement need to be collected in May, at the end of the academic year. The date upon which IRB approval was secured and other time constraints did not allow for such an extended period of data collection. Therefore, without a baseline measure of motivation to learn geometry, analysis of motivation scores was limited to comparisons between the treatment and control group rather than measures of change. Future studies should include an extended period of data collection to obtain pretest and posttest measures of motivation to learn geometry. Also, it is possible the treatment group possessed an awareness of being observed or an awareness that they were in a class that other educators, community members, and members of the media find interesting, thus leading to a Hawthorne effect on the CIS. The experiential GIC course is spotlighted in national, regional, and district level media and public relations outlets, therefore it is possible students enrolled in GIC may have been aware of the interest others have in it as a STEM pedagogical model. Students enrolled in GIC may have exhibited demand characteristics, in which they provided artificially positive responses because they thought that is what is expected of them, when surveyed about the impact GIC had on their motivation to learn geometry (Orne, 1962). Limited interaction between researchers and participants and the administration of the CIS near the completion of the course were attempts to minimize demand characteristics and the Hawthorne effect by limiting the number of times students felt they were being observed by researchers and others.

Conclusion

A quantitative, quasi-experimental research design was used to investigate the effects of an experiential learning course. Analysis of data collected from 181 students experiencing a GIC or traditional geometry curriculum was used to determine the effects of GIC on achievement in geometry and motivation to learn geometry. A convenience sample of all 58 students enrolled in GIC was selected for the treatment group and all 123 students enrolled in traditional geometry was selected for the control group. All participants were secondary students in ninth or tenth grade attending a large, predominantly White, suburban, Midwestern, public high school. The Missouri Geometry EOC Exam was used to measure achievement in geometry. Analysis of covariance (ANCOVA) was used to analyze the effect of group (treatment or control) and gender (male or female) on achievement in geometry while controlling for the covariate of baseline achievement in mathematics as measured by the Missouri Algebra I EOC Exam. Keller's (2010) Course Interest Survey was used to measure motivation to learn geometry within the four subscales of attention, relevance, confidence, and satisfaction as described in Keller's (1987a) ARCS Theory of Motivation. Independent, two-tailed t-tests were used to analyze the effects of group (treatment or control) and gender (male or female) on the motivational sub-components of attention, relevance, confidence, and satisfaction.

CHAPTER 4

FINDINGS

A shortage of interested and qualified workers needed to fill the job openings in STEM and related fields in the U.S. has been reported (Carnevale, Smith, and Strohl, 2010). Attempts to increase the number of students interested and technically prepared to succeed in STEM careers have largely been unsuccessful at meeting the demand (PCAST, 2012). Geometry In Construction (GIC) is an experiential geometry course that provides a transformative model for STEM teaching and learning by utilizing a relevant, service-oriented context for learning. While experiencing the GIC curriculum, students learn and apply principles of geometry, industrial technologies, and career technical education by designing and constructing a small-scale house. The completed house is donated to a local charity serving the needs of homeless people in the community (Contextual Learning Concepts, n.d.a). Chapter four provides the results of research measuring the effects of the GIC curriculum on secondary student achievement in geometry and motivation to learn geometry. Four research questions frame the results.

1. What effect does experiencing the Geometry In Construction curriculum have on the achievement in geometry of secondary students compared to experiencing a traditional geometry curriculum as measured by the Missouri Geometry End of Course Exam?
2. Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?

3. What effect does experiencing the Geometry In Construction curriculum have on the motivation of secondary students to learn geometry compared to experiencing a traditional geometry curriculum as measured by Keller's (2010) Course Interest Survey?
4. Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured Keller's (2010) Course Interest Survey?

Statistical analysis of the quantitative results addresses the null hypotheses developed for each research question by comparing the dependent variable values of the treatment and control groups.

H_0 1: There is no significant difference between the achievement in geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by Missouri Geometry End of Course Exam scores.

H_0 2: Experiencing the Geometry In Construction curriculum does not affect the achievement in geometry of secondary males and females differently as measured by Missouri Geometry End of Course Exam scores.

H_0 3: There is no significant difference between the motivation to learn geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by scores on Keller's (2010) Course Interest Survey.

H_0 4: Experiencing the Geometry In Construction curriculum does not affect the motivation of secondary males and females to learn geometry differently as measured by scores on Keller's (2010) Course Interest Survey.

Data Description

Data were obtained from a sample of 181 ninth and tenth grade students (80 males, 101 females) enrolled in geometry at a large, suburban, Midwestern, public high school. The treatment group consisted of 58 students (35 males, 23 females) enrolled in Geometry In Construction, an experiential geometry course. The control group consisted of 123 students (45 males, 78 females) enrolled in a traditional, lecture-based geometry course. Scores on a Missouri Geometry End of Course Practice Exam were used to measure achievement in geometry. Scores on Keller's (2010) Course Interest Survey (CIS) were used to measure motivation to learn geometry. All data collected were entered into an Excel spreadsheet and checked for errors. Afterward, the data were imported into Statistical Analysis Software (SAS) for descriptive and inferential statistical analysis.

Data Analysis

In the first phase, geometry end of course (EOC) exam scores were collected to determine if there were any differences in achievement in geometry associated with the type of geometry instructional method experienced by students. Geometry EOC exam scores were obtained for 168 participants (33 males and 22 females in the treatment group; 40 males and 73 females in the control group). In order to establish whether the treatment and control groups had similar achievement in mathematics prior to taking geometry, algebra I EOC exam scores were used to determine baseline achievement levels in mathematics. Nearly all of the participants had taken the Missouri Algebra I EOC Exam during the year prior to experiencing a geometry course. Algebra I EOC exam scores were obtained for 177 participants (34 males and 23 females in the treatment group; 44 males and 76 females in the control group).

An independent-samples t-test was conducted to compare the algebra I EOC scores of students enrolled in GIC and students enrolled in traditional geometry. The results of the t-test are shown in Table 6 and indicate that students enrolled in GIC had a significantly higher algebra I EOC score (M = 57.82, SD = 12.77) than students enrolled in traditional geometry (M = 52.78, SD = 13.43), $t(175) = 2.37$, $p = .019$.

Table 6

t-test Results: Comparison of Algebra I EOC Exam Scores of Treatment and Control Groups

	Levene's Test for Equality of Variance		t-test for Equality of Means					95% Confidence Interval of the Difference	
Exam	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Algebra I EOC	1.11	.68	2.37	175	.019	5.04	2.13	0.84	9.24

In addition, simple linear regression of algebra I EOC exam scores against geometry EOC exam scores taken from the whole sample indicated that algebra I EOC exam scores significantly predicted geometry EOC exam scores, $b = 0.70$, $t(163) = 7.39$, $p < .001$. Algebra I EOC exam scores also explained a significant portion of variance in geometry EOC scores, $R^2 = .25$, $F(1,163) = 54.63$, $p < .001$ (see Appendix D). Therefore, algebra I

EOC exam scores were considered to be a covariate, and ANCOVA was chosen as the method to statistically analyze differences in geometry EOC exam scores. A confidence interval of 95% and a type I error rate of .05 were used to interpret all statistical results.

In the second phase, quantitative survey data were collected using John Keller's (2010) Course Interest Survey (CIS) to determine if there were any differences in motivation to learn geometry associated with the type of geometry instructional method experienced by students. Survey data were collected from 95 participants (17 males and 18 females in the treatment group; 31 males and 29 females in the control group) during the last week of their geometry course to measure situational motivation to learn geometry as a result of their geometry course experience. Independent samples, two-tailed t-tests were used to analyze differences in the means of overall motivation scores and motivation subscale scores of attention, relevance, confidence, and satisfaction between the treatment and control groups. These subscales are components of situational motivation as described by Keller's (1987a) ARCS Model which served as the theoretical framework. A confidence interval of 95% and a type I error rate of 0.05 were used to interpret all statistical results.

Results

The following section presents the results for each null hypothesis.

Achievement in Geometry

H_0 1: There is no significant difference between the achievement in geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by Missouri Geometry End of Course Exam scores.

A one-way ANCOVA was conducted to determine if there was a statistically significant difference between the geometry EOC exam scores of students experiencing the GIC curriculum and students experiencing a traditional geometry curriculum when controlling for algebra I EOC exam scores. As depicted in Table 7, the data indicate the geometry EOC exam mean was 12.93 points higher for the treatment group (M = 60.51, SD = 15.79) compared to the control group (M = 47.58, SD = 18.14).

Table 7

Geometry EOC Exam Mean Scores of Treatment and Control Groups

	Geometry EOC Exam Mean Scores	
Group	*M*	*SD*
Treatment (*n* = 55)	60.51	15.79
Control (*n* = 113)	47.58	18.14

Note. *EOC = end of course*. *M* = mean. *SD* = standard deviation. Mean scores indicate the percentage correct.

Table 8 illustrates the results of the ANCOVA which show that experiencing the GIC curriculum had a significant, positive effect on geometry EOC exam scores when controlling for algebra I EOC exam scores, $F(1, 163) = 11.80$, $p < .001$.

Table 8

ANCOVA Results: Comparison of Geometry EOC Exam Scores Between Treatment and Control Groups

Source	*Type III Sum of Squares*	*df*	*Mean Square*	*F*
Course	2818.37	1	2818.37	11.80***
Alg I EOC	10967.74	1	10967.74	45.92***
Error	38454.15	161	238.85	

Note. Algebra I EOC Exam scores are the covariate.

***p < .001

The GIC curriculum had an intermediate effect size on geometry EOC exam scores as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.60$). "Hedges' "*g*" is an appropriate measure of effect size for mean differences of groups with unequal sample sizes within a pre- post- control design" (Morris, 2008). Interpretations of effect sizes vary in the literature, but Lenhard and Lenhard (2016) suggested the range 0.2 to 0.4 is a small effect, 0.5 to 0.7 is an intermediate effect, and above 0.8 is a large effect.

H_0 2: Experiencing the Geometry In Construction curriculum does not affect the achievement in geometry of secondary males and females differently as measured by Missouri Geometry End of Course Exam scores.

In order to determine if the GIC curriculum had a significantly different effect on the achievement in geometry of males and females, three research models corresponding to research question two were created, as illustrated in Table 9. The first model, 2A, compared the geometry EOC exam scores of males in GIC to females in GIC. The second model, 2B, compared the geometry EOC exam scores of males in GIC to males in traditional geometry. The third model, 2C, compared the geometry EOC exam scores of females in GIC to females in traditional geometry.

Table 9

ANCOVA Research Models for Analyzing Achievement in Geometry Based on Gender

Model	Research Question	Independent Variable	Dependent Variable	Covariate	Comparison Group
2A	Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?	gender	achievement in geometry	Algebra I EOC Exam Scores	GIC males compared to GIC females
2B	Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?	curriculum	achievement in geometry	Algebra I EOC Exam Scores	GIC males compared to control males
2C	Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?	curriculum	achievement in geometry	Algebra I EOC Exam Scores	GIC females compared to control females

Achievement in Geometry: Treatment Group Gender Comparison

For research model 2A (see Table 9), a one-way ANCOVA was conducted to determine if there was a statistically significant difference between the geometry EOC exam scores of males experiencing the GIC curriculum and females experiencing the GIC curriculum when controlling for algebra I EOC exam scores. As depicted in Table 10, the data indicate the geometry EOC exam mean score was 2.06 points higher for males in the treatment group (M = 61.33, SD = 16.61) compared to females in the treatment group (M = 59.27, SD = 14.75).

Table 10

Geometry EOC Exam Mean Scores of Treatment Males Compared to Treatment Females

	Geometry EOC Exam Mean Scores	
Group	*M*	*SD*
Treatment Males (n = 35)	61.33	16.61
Treatment Females (n = 23)	59.27	14.75

Note. *EOC = end of course*. *M* = mean. *SD* = standard deviation. Mean scores indicate the percentage correct.

Table 11 illustrates the results of the ANCOVA for research model 2A which show that gender did not have a significant effect on geometry EOC exam scores of students in the GIC course when controlling for algebra I EOC exam scores, $F(1, 53) = .49$, $p = .486$. Gender had an insignificant, small effect size on achievement in geometry within the treatment group as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.22$).

Table 11

ANCOVA Results: Achievement in Geometry of Treatment Males Compared to Treatment Females

Source	*Type III Sum of Squares*	*df*	*Mean Square*	*F*
Gender	113.81	1	113.81	0.49
Alg I EOC	1586.21	1	1586.21	6.85*
Error	11802.03	51	231.41	

Note. Algebra I EOC Exam scores are the covariate.

*p < .05

Achievement in Geometry: Treatment Males Compared to Control Males

For research model 2B (see Table 9), a one-way ANCOVA was conducted to determine if there was a statistically significant difference between the geometry EOC exam scores of males experiencing the GIC curriculum and males experiencing a traditional geometry curriculum when controlling for algebra I EOC exam scores. As depicted in Table 12, the data indicate the geometry EOC exam mean score was 8.90 points higher for males in the treatment group (M = 61.33, SD = 16.61) compared to males in the control group (M = 52.43, SD = 18.92).

Table 12

Geometry EOC Exam Mean Scores of Treatment Males Compared to Control Males

	Geometry EOC Exam Mean Scores	
Group	*M*	*SD*
Treatment Males (n = 35)	61.33	16.61
Control Males (n = 45)	52.43	18.92

Note. *EOC = end of course*. *M* = mean. *SD* = standard deviation. Mean scores indicate the percentage correct.

Table 13 illustrates the results of the ANCOVA for research model 2B which show that the GIC curriculum did not have a significant effect on geometry EOC exam scores of males in the treatment group compared to males in the control group when controlling for algebra I EOC exam scores, $F(1, 70) = 2.84$, $p = .097$. In the comparison of treatment males to control males, the GIC curriculum had an insignificant, intermediate effect size on achievement in geometry as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.57$).

Table 13

ANCOVA Results: Achievement in Geometry of Treatment Males Compared to Control Males

Source	*Type III Sum of Squares*	*df*	*Mean Square*	*F*
Course	774.78	1	774.78	2.84
Alg I EOC	3894.11	1	3894.11	14.25***
Error	18578.13	68	273.21	

Note. Algebra I EOC Exam scores are the covariate.

***$p < .001$

Achievement in Geometry: Treatment Females Compared to Control Females

For research model 2C (see Table 9), a one-way ANCOVA was conducted to determine if there was a statistically significant difference between the geometry EOC exam scores of females experiencing the GIC curriculum and females experiencing a traditional geometry curriculum when controlling for algebra I EOC exam scores. As depicted in Table 14, the data indicate the geometry EOC exam mean score was 14.34 points higher for females in the treatment group (*M* = 59.27, *SD* = 14.75) compared to females in the control group (*M* = 44.93, *SD* = 17.25).

Table 14

Geometry EOC Exam Mean Scores of Treatment Females Compared to Control Females

	Geometry EOC Exam Mean Scores	
Group	*M*	*SD*
Treatment Females (*n* = 23)	59.27	14.75
Control Females (*n* = 78)	44.93	17.25

Note. *EOC = end of course. M* = mean. *SD* = standard deviation. Mean scores indicate the percentage correct.

Table 15 illustrates the results of the ANCOVA for research model 2C which show that the GIC curriculum had a significant, positive effect on geometry EOC exam scores of females in the treatment group compared to females in the control group when controlling for algebra I EOC exam scores, $F(1, 92) = 6.32$, $p = .014$. In the comparison of treatment females to control females, the GIC curriculum had an intermediate effect size on achievement in geometry as determined by calculating Hedges' "*g*" ($g_{Hedges'}$ = 0.52).

Table 15

ANCOVA Results: Achievement in Geometry of Treatment Females Compared to Control Females

Source	*Type III Sum of Squares*	*df*	*Mean Square*	*F*
Course	1326.42	1	1326.42	6.32*
Alg I EOC	6533.23	1	6533.23	31.14***
Error	18883.53	90	209.82	

Note. Algebra I EOC Exam scores are the covariate.

*p < .05. ***p < .001

A summary of ANCOVA results indicating significant differences in geometry EOC exam mean scores and corresponding to research questions one and two is shown in Table 16.

Table 16

Summary of ANCOVA Results: Significant Differences in Achievement in Geometry

Result	**RQ1** GIC students (*n*=55) compared to TG students (*n*=113)	**RQ2A** GIC males (*n*=33) compared to GIC females (*n*=22)	**RQ2B** GIC males (*n*=33) compared to TG males (*n*=40)	**RQ2C** GIC females (*n*=22) compared to TG females (*n*=73)
Significance	*** p = .0008	p = .4863	p = .0968	* p = .0137
Effect Size	intermediate $g_{Hedges'}$ = 0.60	small $g_{Hedges'}$ = 0.22	intermediate $g_{Hedges'}$ = 0.57	intermediate $g_{Hedges'}$ = 0.52

Note. RQ2A, RQ2B, and RQ2C are research models corresponding to research question two (See Table 9). EOC = end of course. GIC = Geometry In Construction treatment group. TG = traditional geometry control group.

*p < .05. ***p < .001.

Motivation to Learn Geometry

***H_0* 3: There is no significant difference between the motivation to learn geometry of secondary students experiencing the Geometry In Construction curriculum and those experiencing a traditional geometry curriculum as measured by scores on Keller's (2010) Course Interest Survey.**

Table 17 shows a comparison of the motivation to learn geometry mean scores for students in the treatment and control group. The scores include an overall motivation score and subscale scores for attention, relevance, confidence, and satisfaction as described previously by Keller's (1987a) ARCS motivation model.

Table 17

Motivation to Learn Geometry Mean Scores of Treatment and Control Groups

	Overall Motivation		Attention		Relevance		Confidence		Satisfaction	
Group	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Treatment Group (*n* = 35)	3.62	0.75	3.12	0.88	3.56	0.86	4.16	0.59	3.64	0.96
Control Group (*n* = 60)	2.97	0.69	2.38	0.79	3.02	0.69	3.54	0.85	2.94	0.87

Note. M = mean. SD = standard deviation. Treatment = Geometry in Construction students.

Control = traditional geometry students. Mean scores are based on a five-point scale.

T-tests were conducted to determine if there were statistically significant differences between the motivation mean scores of students experiencing the GIC curriculum and students experiencing a traditional geometry curriculum. Table 18 shows the t-test results.

Table 18.

t-test Results: Comparison of Motivation to Learn Geometry Mean Scores of Treatment to Control Groups

	GIC students (treatment)		traditional geometry students (control)			
Scale	*M*	*SD*	*M*	*SD*	*t-test*	*df*
Overall Motivation	3.62	0.75	2.97	0.69	4.24***	93
Attention	3.12	0.88	2.38	0.79	4.24***	93
Relevance	3.56	0.86	3.02	0.69	3.37**	93
Confidence	4.16	0.59	3.54	0.85	3.77***	93
Satisfaction	3.64	0.96	2.94	0.87	3.67***	93

Note. M = mean. SD = standard deviation. Mean scores are based on a five-point scale.

$^{**}p < .01$. $^{***}p < .001$.

The results indicate the following:

Overall Motivation: Treatment Compared to Control

- Geometry In Construction students had a 0.65-point higher overall motivation mean score (*M* = 3.62, *SD* = 0.75) compared to traditional geometry students (*M* = 2.97, *SD* = 0.69), and that difference was significant, $t(93) = 4.24$, $p < .001$.
- The overall motivation scores of the treatment group were skewed right and nonparametric (see Appendix F), therefore, a Wilcoxon Rank-Sum Test was also used to confirm a significant difference between the means ($p < .001$).
- The GIC curriculum had a large effect size on overall motivation scores as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.90$).

Attention Subscale: Treatment Compared to Control

- Geometry In Construction students had a 0.74-point higher attention mean score (*M* = 3.12, *SD* = 0.88) compared to traditional geometry students (*M* = 2.38, *SD* = 0.79), and that difference was significant, $t(93) = 4.24$, $p < .001$.
- The GIC curriculum had a large effect size on attention scores as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.90$).

Relevance Subscale: Treatment Compared to Control

- Geometry In Construction students had a 0.54-point higher relevance mean score (*M* = 3.56, *SD* = 0.86) compared to traditional geometry students (*M* = 3.02, *SD* = 0.69), and that difference was significant $t(93) = 3.37$, $p = .001$.
- The GIC curriculum had an intermediate effect size on relevance scores as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.72$).

Confidence Subscale: Treatment Compared to Control

- Geometry In Construction students had a 0.62-point higher confidence mean score (M = 4.16, SD = 0.59) compared to traditional geometry students (M = 3.54, SD = 0.85), and that difference was significant, $t(93) = 3.77$, $p < .001$.
- The confidence scores of both the treatment and control groups were nonparametric (see Appendix F), therefore, a Wilcoxon Rank-Sum Test was also used to confirm a significant difference between the means ($p < .001$).
- The GIC curriculum had a large effect size on confidence scores as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.80$).

Satisfaction Subscale: Treatment Compared to Control

- Geometry In Construction students had a 0.70-point higher satisfaction mean score (M = 3.64, SD = 0.96) compared to traditional geometry students (M = 2.94, SD = 0.87), and that difference was significant, $t(93) = 3.67$, $p < .001$.
- The GIC curriculum had an intermediate effect size on satisfaction scores as determined by calculating Hedges' "*g*" ($g_{Hedges'} = 0.78$).

A summary of these results is presented in Table 24.

H_0 4: Experiencing the Geometry In Construction curriculum does not affect the motivation of secondary males and females to learn geometry differently as measured by scores on Keller's (2010) Course Interest Survey.

In order to determine if the GIC curriculum affected the motivation of males and females to learn geometry differently, three research models corresponding to research question four were created, as illustrated in Table 19. The first model, 4A, compared the

Table 19

t-test Research Models for Analyzing Motivation to Learn Geometry Based on Gender

Model	Research Question	Independent Variable	Dependent Variable(s)	Comparison Group
4A	Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?	gender	motivation attention relevance confidence satisfaction	GIC males compared to GIC females
4B	Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?	curriculum	motivation attention relevance confidence satisfaction	GIC males compared to control males
4C	Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?	curriculum	motivation attention relevance confidence satisfaction	GIC females compared to control females

Table 20

Motivation to Learn Geometry Mean Scores of Treatment and Control Groups by Gender

	Overall Motivation		Attention		Relevance		Confidence		Satisfaction	
Group	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Treatment Males (n = 17)	3.55	0.73	2.84	0.83	3.50	0.91	4.29	0.48	3.58	1.02
Treatment Females (n = 18)	3.68	0.79	3.39	0.86	3.62	0.84	4.03	0.67	3.70	0.92
Control Males (n = 31)	3.14	0.65	2.55	0.77	3.16	0.74	3.79	0.66	3.05	0.78
Control Females (n = 29)	2.79	0.71	2.20	0.78	2.87	0.61	3.27	0.95	2.82	0.96

Note. M = mean. SD = standard deviation. Treatment = Geometry in Construction students. Control = traditional geometry students. Mean scores are based on a five-point scale.

motivation scores of males in GIC to females in GIC. The second model, 4B, compared the motivation scores of males in GIC to males in traditional geometry. The third model, 4C, compared the motivation scores of females in GIC to females in traditional geometry. Table 20 shows a comparison of the motivation mean scores of the treatment and control groups separated by gender.

For research model 4A (see Table 19), t-tests were conducted to determine if there were statistically significant differences between the motivation mean scores of males experiencing the GIC curriculum and females experiencing the GIC curriculum. Table 21 illustrates the t-test results for research model 4A which show that gender did not have a significant effect on any of the motivation mean scores of students in the GIC course.

Table 21

t-test Results: Comparison of Motivation to Learn Geometry Mean Scores of Treatment Males to Treatment Females

	GIC males		GIC females			
Scale	*M*	*SD*	*M*	*SD*	*t-test*	*df*
Overall Motivation	3.55	0.73	3.68	0.79	-0.48	33
Attention	2.84	0.83	3.39	0.86	-1.94	33
Relevance	3.50	0.91	3.62	0.84	-0.38	33
Confidence	4.29	0.48	4.03	0.67	1.35	33
Satisfaction	3.58	1.02	3.70	0.92	-0.37	33

Note. M = mean. SD = standard deviation. GIC = Geometry In Construction.

Means are based on a five-point scale.

The following is a further depiction of the results:

Overall Motivation: Treatment Group Gender Comparisons

- Males in the treatment group had a 0.13-point lower overall motivation mean score (M = 3.55, SD = 0.73) compared to females in the treatment group (M = 3.68, SD = 0.79), but the difference was not significant, $t(33) = -0.48$, $p = .632$.
- The overall motivation scores of females in the treatment group were skewed right and nonparametric (see Appendix F), therefore, a Wilcoxon Rank-Sum Test was also used to confirm that the difference between the means was not significant ($p = .680$).
- Gender had an insignificant, small effect size on overall motivation scores within the treatment group as determined by calculating Hedges' "*g*" ($g = 0.16$).

Attention Subscale: Treatment Group Gender Comparisons

- Males in the treatment group had a 0.55-point lower attention mean score (M = 2.84, SD = 0.83) compared to females in the treatment group (M = 3.39, SD = 0.86), but the difference was not significant, $t(33) = -1.94$, $p = .061$.
- Gender had an insignificant, intermediate effect size on attention scores within the treatment group as determined by calculating Hedges' "*g*" ($g = 0.66$).

Relevance Subscale: Treatment Group Gender Comparisons

- Males in the treatment group had a 0.12-point lower relevance mean score (M = 3.50, SD = 0.91) compared to females in the treatment group (M = 3.62, SD = 0.84), but the difference was not significant, $t(33) = -0.38$, $p = .703$.

- Gender had an insignificant, small effect size on relevance scores within the treatment group as determined by calculating Hedges' "*g*" (g = 0.13).

Confidence Subscale: Treatment Group Gender Comparisons

- For the confidence subcomponent of motivation, males in the treatment group had a 0.26-point higher mean score (M = 4.29, SD = 0.48) compared to females in the treatment group (M = 4.03, SD = 0.67), but the difference was not significant, $t(33)$ = 1.35, p =.186.
- The confidence scores of females in the treatment group were skewed right and nonparametric (see Appendix F), therefore, a Wilcoxon Rank-Sum Test was also used to confirm that the difference between the means was not significant (p = .379).
- Gender had an insignificant, intermediate effect size on confidence scores within the treatment group as determined by calculating Hedges' "*g*" (g = 0.46).

Satisfaction Subscale: Treatment Group Gender Comparisons

- Males in the treatment group had a 0.12-point lower satisfaction mean score (M = 3.58, SD = 1.02) compared to females in the treatment group (M = 3.70, SD = 0.92), but the difference was not significant, $t(33)$ = -0.37, p =.714.
- Gender had an insignificant, small effect size on satisfaction scores as determined by calculating Hedges' "*g*" (g = 0.13).

  A summary of these results is presented in Table 24.

For research model 4B (see Table 19), t-tests were conducted to determine if there were statistically significant differences between the motivation mean scores of males experiencing the GIC curriculum and males experiencing the traditional geometry curriculum. As shown in Table 22, the results of the t-tests were mixed.

Table 22

t-test Results: Comparison of Motivation to Learn Geometry Mean Scores of Treatment Males to Control Males

	GIC males (treatment)		traditional geometry males (control)			
Scale	*M*	*SD*	*M*	*SD*	*t-test*	*df*
Overall Motivation	3.55	0.73	3.14	0.65	2.02*	46
Attention	2.84	0.83	2.55	0.77	1.21	46
Relevance	3.50	0.91	3.16	0.74	1.41	46
Confidence	4.29	0.48	3.79	0.66	2.75**	46
Satisfaction	3.58	1.02	3.05	0.78	2.04*	46

Note. M = mean. SD = standard deviation. GIC = Geometry In Construction.

Mean scores based on five-point scale.

*p < .05. **p < .01.

The following is a further depiction of the results:

Overall Motivation: Treatment Males Compared to Control Males

- Males in the treatment group had a 0.41-point higher overall motivation mean score (M = 3.55, SD = 0.73) compared to males in the control group (M = 3.14, SD =0.65), and the difference was significant, t(46) = 2.02, p = .049.
- In the comparison of treatment males to control males, the GIC curriculum had a significant, intermediate effect size on overall motivation scores as determined by calculating Hedges' "g" (g = 0.61).

Attention Subscale: Treatment Males Compared to Control Males

- Males in the treatment group had a 0.29-point higher attention mean score (M = 2.84, SD = 0.83) compared to males in the control group (M = 2.55, SD = 0.77), but the difference was not significant, $t(46) = 1.21$, $p = .232$.
- In the comparison of treatment males to control males, the GIC curriculum had an insignificant, small effect size on attention scores as determined by calculating Hedges' "*g*" (g = 0.37).

Relevance Subscale: Treatment Males Compared to Control Males

- Males in the treatment group had a 0.34-point higher relevance mean score (M = 3.50, SD = 0.91) compared to males in the control group (M = 3.16, SD = 0.74), but the difference was not significant, $t(46) = 1.41$, $p = .166$.
- In the comparison of treatment males to control males, the GIC curriculum had an insignificant, intermediate effect size on relevance scores as determined by calculating Hedges' "*g*" (g = 0.43).

Confidence Subscale: Treatment Males Compared to Control Males

- Males in the treatment group had a 0.50-point higher confidence mean score (M = 4.29, SD = 0.48) compared to males in the control group (M = 3.79, SD = 0.66), and the difference was significant, $t(46) = 2.75$, $p = .009$.
- In the comparison of treatment males to control males, the GIC curriculum had a significant, large effect size on confidence scores as determined by calculating Hedges' "*g*" (g = 0.83).

Satisfaction Subscale: Treatment Males Compared to Control Males

- Males in the treatment group had a 0.53-point higher satisfaction mean score (M = 3.58, SD = 1.02) compared to males in the control group (M = 3.05, SD = 0.78), and the difference was significant, $t(46) = 2.04$, p =.048.
- In the comparison of treatment males to control males, the GIC curriculum had a significant, intermediate effect size on satisfaction scores as determined by calculating Hedges' "*g*" (g = 0.62).

A summary of these results is presented in Table 24.

For research model 4C (see Table 19), t-tests were conducted to determine if there were statistically significant differences between the motivation mean scores of females experiencing the GIC curriculum and females experiencing the traditional geometry curriculum. As shown in Table 23, the GIC curriculum had a significant, large, positive effect on every measure of motivation for females.

Table 23

t-test Results: Comparison of Motivation to Learn Geometry Mean Scores of Treatment Females to Control Females

	GIC females (treatment)		traditional geometry females (control)			
Scale	*M*	*SD*	*M*	*SD*	*t-test*	*df*
Overall Motivation	3.68	0.79	2.79	0.71	3.99***	45
Attention	3.39	0.86	2.20	0.78	4.90***	45
Relevance	3.62	0.84	2.87	0.61	3.55***	45
Confidence	4.03	0.67	3.27	0.95	2.93**	45
Satisfaction	3.70	0.92	2.82	0.96	3.10**	45

Note. M = mean. SD = standard deviation. GIC = Geometry In Construction

Mean scores based on five-point scale.

**p < .01. ***p < .001.

The following is a further depiction of the results:

Overall Motivation: Treatment Females Compared to Control Females

- Females in the treatment group had a 0.89-point higher overall motivation mean score (M = 3.68, SD = 0.79) compared to females in the control group (M = 2.79, SD = 0.71), and the difference was significant, $t(45) = 3.99$, $p < .001$.
- The overall motivation scores of females in the treatment group were skewed right and nonparametric (see Appendix F), therefore, a Wilcoxon Rank-Sum Test was also used to confirm a significant difference between the means ($p < .001$).
- In the comparison of treatment females to control females, the GIC curriculum had a significant, large effect size on overall motivation scores as determined by calculating Hedges' "*g*" ($g = 1.20$).

Attention Subscale: Treatment Females Compared to Control Females

- Females in the treatment group had a 1.19-point higher attention mean score (M = 3.39, SD = 0.86) compared to females in the control group (M = 2.20, SD = 0.78), and the difference was significant, $t(45) = 4.90$, $p < .001$.
- In the comparison of treatment females to control females, the GIC curriculum had a significant, large effect size on attention scores as determined by calculating Hedges' "*g*" ($g = 1.47$).

Relevance Subscale: Treatment Females Compared to Control Females

- Females in the treatment group had a 0.75-point higher relevance mean score (M = 3.62, SD = 0.84) compared to control females (M = 2.87, SD = 0.61), and the difference was significant, $t(45) = 3.55$, $p < .001$.

- In the comparison of treatment females to control females, the GIC curriculum had a significant, large effect size on relevance scores as determined by calculating Hedges' "*g*" (*g* = 1.07).

Confidence Subscale: Treatment Females Compared to Control Females

- Females in the treatment group had a 0.76-point higher confidence mean score (*M* = 4.03, *SD* = 0.67) compared to females in the control group (*M* = 3.27, *SD* = 0.95), and the difference was significant, t(45) = 2.93, *p* =.005.
- The confidence scores of females in both the treatment and control groups were nonparametric (see Appendix F), therefore, a Wilcoxon Rank-Sum Test was also used to confirm a significant difference between the means ($p < .004$).
- In the comparison of treatment females to control females, the GIC curriculum had a significant, large effect size on confidence scores as determined by calculating Hedges' "*g*" (*g* = 0.88).

Satisfaction Subscale: Treatment Females Compared to Control Females

- Females in the treatment group had a 0.88-point higher satisfaction mean score (*M* = 3.70, *SD* = 0.92) compared to females in the control group (*M* = 2.82, *SD* = 0.96), and the difference was significant, *t*(45) = 3.10, *p* =.003.
- In the comparison of treatment females to control females, the GIC curriculum had a significant, large effect size on satisfaction scores as determined by calculating Hedges' "*g*" (*g* = 0.93).

Summary

A summary of all t-test results indicating significant differences in motivation and corresponding to research questions three and four is presented in Table 24.

Table 24

Summary of t-test Results: Significant Differences in Motivation to Learn Geometry

	RQ3 GIC students (n=35) compared to TG students (n=60)	RQ4A GIC males (n=17) compared to GIC females (n=18)	RQ4B GIC males (n=17) compared to TG males (n=31)	RQ4C GIC females (n=18) compared to TG females (n=29)
Overall Motivation	*** *p* = .0001 $g_{Hedges'}$ = 0.90 large ES	*p* = .6797 $g_{Hedges'}$ = 0.16 small ES on females	* *p* = .0494 $g_{Hedges'}$ = 0.61 intermediate ES	*** *p* = .0005 $g_{Hedges'}$ = 1.20 large ES
Attention	*** *p* = .0001 $g_{Hedges'}$ = 0.90 large ES	*p* = .0608 $g_{Hedges'}$ = 0.66 intermediate ES on females	*p* = .2316 $g_{Hedges'}$ = 0.37 small ES	*** *p* = .0001 $g_{Hedges'}$ = 1.47 large ES
Relevance	** *p* = .0011 $g_{Hedges'}$ = 0.72 intermediate ES	*p* = .7028 $g_{Hedges'}$ = 0.13 small ES on females	*p* = .1658 $g_{Hedges'}$ = 0.43 intermediate ES	*** *p* = .0009 $g_{Hedges'}$ = 1.07 large ES
Confidence	*** *p* = .0004 $g_{Hedges'}$ = 0.80 large ES	*p* = .3790 $g_{Hedges'}$ = 0.46 intermediate ES on males	** *p* = .0085 $g_{Hedges'}$ =0 .83 large ES	** *p* = .0074 $g_{Hedges'}$ = 0.88 large ES
Satisfaction	*** *p* = .0004 $g_{Hedges'}$ = 0.78 intermediate ES	*p* = .7143 $g_{Hedges'}$ = 0.13 small ES on females	* *p* = .0475 $g_{Hedges'}$ = 0.62 intermediate ES	** *p* = .0033 $g_{Hedges'}$ = 0.93 large ES

Note. RQ4A, RQ4B, and RQ4C are research models corresponding to research question four (see Table 19 for details). GIC = Geometry In Construction treatment group. TG = Traditional geometry control group. ES = Hedges' "g" effect size.

*p < .05. **p < .01. ***p < .001.

CHAPTER 5

CONCLUSION

Some schools are following the suggestion of the President's Council on the Advancement of Science and Technology to transform STEM teaching and learning by exploring innovative teaching models and curricula. Some models include theme based experiential courses that provide career-oriented experiences where students engage in curriculum related to the professional industry and develop 21st century skills, such as collaboration, problem solving, and communication (Center for Advanced Professional Studies, n.d.). These models attempt to address the lack of interest and motivation students have for pursuing STEM careers as noted by Wang, Eccles, and Kenny (2013).

In the investigation of an innovative, experiential geometry course called Geometry In Construction (GIC), in a suburban high school, four hypotheses were explored. While experiencing the GIC curriculum, students learned and applied principles of geometry, industrial technologies, and career technical education by designing and constructing a small-scale house. The completed house was donated to a local charity serving the needs of homeless people in the community (Contextual Learning Concepts, n.d.a). The empirical evidence discussed in this chapter justifies the decision to explore or implement future experiential models.

Summary of Findings

An investigation was conducted in order to compare the achievement and motivation in geometry of secondary students completing an experiential learning course to those completing a traditional geometry course. Table 25 summarizes the findings for each research question.

Table 25

Summary of Findings

Question	Findings
In response to research question 1: *What effect does experiencing the Geometry In Construction curriculum have on the achievement in geometry of secondary students compared to experiencing a traditional geometry curriculum as measured by the Missouri Geometry End of Course Exam?*	• Students who were taught using the experiential GIC curriculum demonstrated significantly higher achievement in geometry than students who were taught using the traditional geometry curriculum.
In response to research question 2*:* *Does experiencing the Geometry In Construction curriculum affect the achievement in geometry of secondary males and females differently as measured by the Missouri Geometry End of Course Exam?*	• Females who were taught using the experiential GIC curriculum demonstrated significantly higher achievement in geometry than females who were taught using the traditional geometry curriculum. • There was no significant difference between the achievement in geometry of males who experienced the GIC curriculum and males who experienced the traditional geometry curriculum. • There was no significant difference between the achievement in geometry of males and females who both experienced the GIC curriculum.
In response to research question 3: *What effect does experiencing the Geometry In Construction curriculum have on the motivation of secondary students to learn geometry compared to experiencing a traditional geometry curriculum as measured by Keller's (2010) Course Interest Survey?*	• Students who were taught using the experiential GIC curriculum reported significantly higher overall motivation to learn geometry than students who were taught using the traditional geometry curriculum. ○ GIC had a large effect on motivation subscales measuring attention and confidence. ○ GIC had an intermediate effect on motivation subscales measuring relevance and satisfaction.
In response to research question 4: *Does experiencing the Geometry In Construction curriculum affect the motivation of secondary males and females to learn geometry differently as measured by Keller's (2010) Course Interest Survey?*	• The experiential GIC curriculum affected the overall motivation of males and females to learn geometry differently when compared to same gender peers taught using the traditional geometry curriculum. ○ Female students who were taught using the experiential GIC curriculum reported significantly higher motivation to learn geometry on subscales measuring attention, relevance, confidence and satisfaction than females who were taught using the traditional geometry curriculum. ○ Male students who were taught using the experiential GIC curriculum reported significantly higher motivation to learn geometry on subscales measuring confidence and satisfaction than males who were taught using the traditional geometry curriculum. • There was no significant difference in motivation to learn geometry between males and females who both experienced the GIC curriculum.

Generalizations based on the findings should be limited to:

- similar populations of Midwestern, suburban, predominantly White, public secondary schools,
- settings with experienced geometry teachers,
- post-treatment comparisons of achievement and motivation in geometry between treatment and control groups rather than growth in achievement and motivation.

Conclusions

The most prominent findings to emerge from the study were those describing the surprisingly larger effect the experiential curriculum had on female achievement and motivation to learn geometry. The findings led to three notable conclusions discussed in this section.

1. Females completing the year-long experiential geometry curriculum reported much higher overall motivation, attention, relevance, confidence and satisfaction associated with learning geometry than females completing the traditional geometry curriculum.

2. Females completing the year-long experiential geometry curriculum demonstrated higher achievement in geometry than females completing the traditional curriculum.

3. There were no significant differences in achievement in geometry amongst males and females who both completed the year-long experiential geometry curriculum.

Discussion and Implications

Before discussing the conclusions and implications, it is important to highlight the literature that identifies several factors impacting the motivation of females to pursue STEM learning and careers. The discussion elaborates on how the findings connect to the existing knowledge base in the literature.

Women in STEM

While lack of motivation and interest deters both male and female high school students from pursuing STEM, Wang et al. (2013) described it as the primary factor impeding female representation in certain STEM professions. Low interest in high school mathematics coursework decreases the likelihood that females will choose a post-secondary STEM major (Blickenstaff, 2005). Several studies found lack of confidence, influenced by environmental factors, to be the root cause of low interest in mathematics among females (Beilock etal., 2010; Eccles & Wang, 2016; Ganley & Lubienski, 2016). In addition, motivation to learn mathematics appeared in the literature to be a strong indicator for matriculation into STEM, especially among females (Eccles & Wang, 2016; PCAST, 2012). Therefore, an innovative, experiential learning course was investigated in order to determine the effects it had on achievement in geometry and motivation to learn geometry. The results of the investigation led to the following conclusions.

Conclusion 1: Females who completed the year-long experiential geometry curriculum reported significantly higher motivation to learn geometry than females who completed the traditional geometry curriculum.

There are various theories that explain factors leading to the motivation of K-12 students. John Keller's (2010) ARCS theory of motivation was used to identify specific motivational factors influencing secondary students situated in experiential and traditional geometry classrooms. According to Keller (1987a), four personal attributes must be addressed to initiate and sustain learner motivation: attention, relevance, confidence and satisfaction. At the end of the school year, males and females in both the experiential and traditional geometry classrooms completed Keller's (2010) Course Interest Survey (see Appendix C) which measured the effect of the instructional methods, materials and conditions on their motivation to learn geometry. All participants were asked to respond to 34 statements in order to measure their attention, relevance, confidence and satisfaction. Students indicated how "true" the statements were in relation to their geometry course: not true, slightly true, moderately true, mostly true, and very true. For example, one statement read, "I feel confident that I will do well in my math class". Both males and females in the experiential geometry course reported higher levels of motivation compared to males and females in the traditional course (see Figure 9). However, the most interesting findings were associated with how much larger the motivation scores were for females in the experiential geometry course compared to females in the traditional geometry course (see Figure 10).

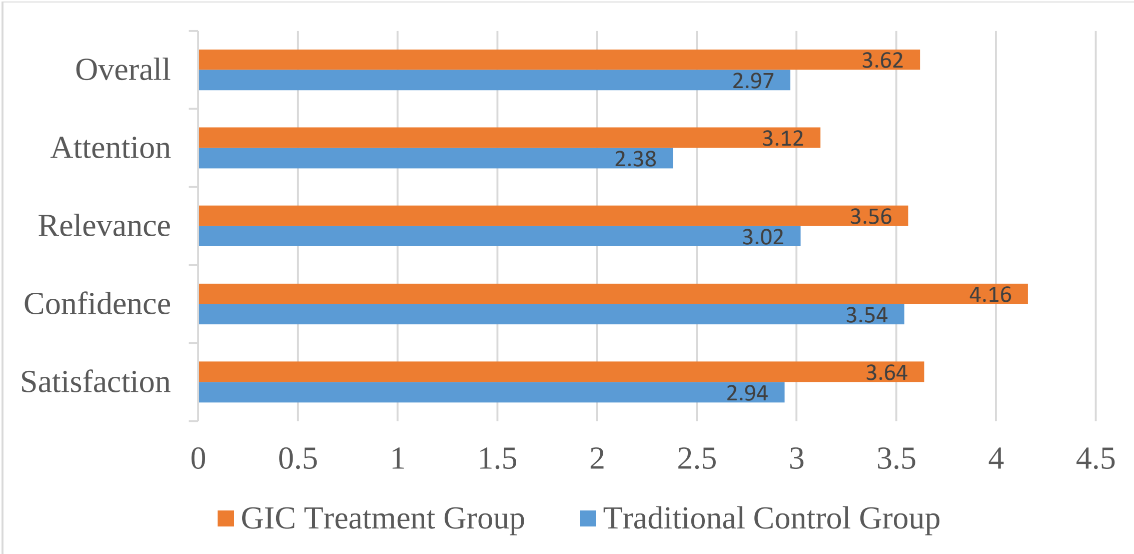

Figure 9. Motivation scores of treatment and control groups.

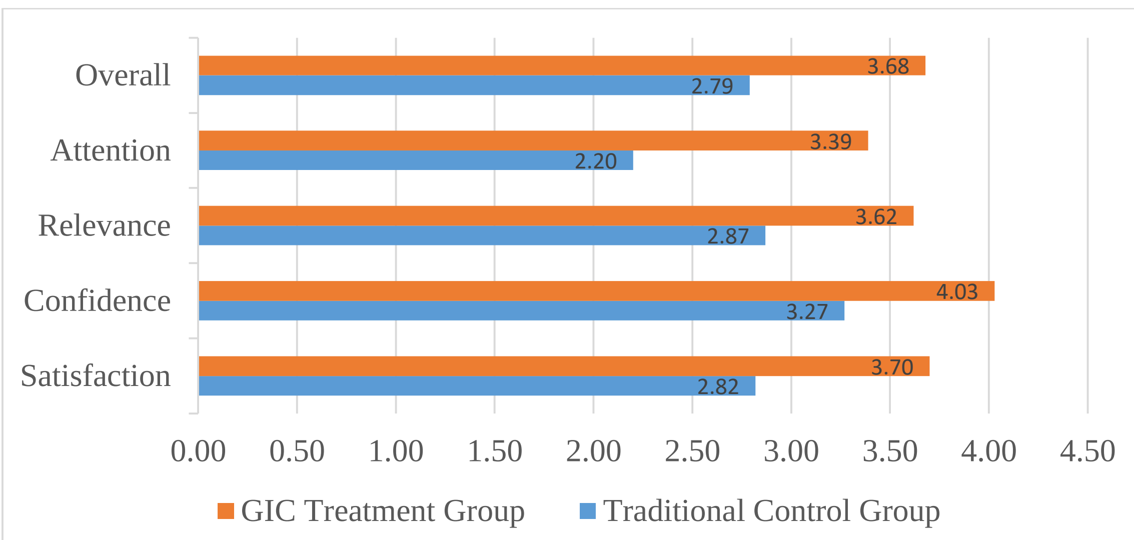

Figure 10. Motivation scores of females.

Females in the GIC course reported significantly higher motivation on all four of Keller's motivational subscales (attention, relevance, confidence, and satisfaction). Because several studies suggested that environmental and contextual factors indirectly affect female performance and interest in mathematics, the results can be used to identify specific classroom conditions that contributed to the significant differences in motivation to learn geometry (Ganley & Lubienski, 2106; Lindberg, Hyde, Peterson, & Linn, 2010; Andreescu et al. 2008). Such conditions might be replicated in other mathematics strands to increase female interest in mathematics and STEM. With the absence of qualitative data, only knowledge of the GIC instructional model and supporting research can be used to predict which aspects of the experiential curriculum may have contributed to this conclusion.

Attention and Relevance

The design of the GIC course includes various instructional approaches supporting the findings related to attention and relevance. Students experiencing the GIC curriculum were engaged in a service-learning project where they applied learned principles to design and construct a real house for homeless community members. The active experimentation and complexity of building the house attributes to what Keller (1987a) called inquiry arousal which often captures the attention of the learner. In addition, service-learning experiences often foster a sense of civic responsibility (National Service-Learning Clearinghouse, 2013), thus prolonging attention over a period of time, which Keller (1987a) references as another key factor attributing to learner motivation.

From the perspective of Dewey (2008), the tangible service-learning experience also contributed to the reported sense of relevancy as the acquired knowledge and skills were directly applied to effect a societal change. The findings regarding the significantly increased motivation of females in the treatment group provide strong evidence supporting the inclusion of a humanistic approach in mathematics curricula. Eccles &Wang (2016) proposed that females value STEM occupations involving human interactions and are less attracted to fields focused on mathematics. Perhaps the heightened sense of relevancy reported by females taking the GIC course may be attributed to the positive effect their learning and work had on other humans. This would be supported by the work of Keller (1987a) who related relevance to the ability of students to connect their learning experience (building a house) to a personal need or value (interactions and care for humans). The realization by students that mathematics is relevant and aligns with their personal values could have a positive impact on their desire to pursue careers in STEM.

The results provide evidence that an experiential learning geometry curriculum, emphasizing a humanistic approach, significantly improved attention and made learning geometry more relevant for males and females. The large effect size which the GIC curriculum had on female attention and relevance was surprising. These findings make a strong case for considering a similar approach in other mathematics strands as a motivational construct to improve interest and preparation for further STEM coursework and careers.

Confidence

Females in the GIC course reported significantly higher confidence associated with learning geometry compared to females in a traditional geometry course. Females in the experiential learning course were presented daily opportunities to build confidence and reduce the anxiety associated with learning geometry as they worked in small teams to solve problems associated with designing and building a house. They interacted in an environment where their ideas were equally valued and contributed to the success of a larger goal. Keller's (1987a) motivational model delineates the acquired confidence of the females as stemming from their ability to link their success to their effort exerted while building a house. This finding is especially important for females, as Ganley and Lubienski (2016) reported confidence towards mathematics to be the largest contextual concern indirectly affecting performance in this discipline.

By middle-school, we begin to see the unintended consequence from the persistent use of a recitation model to teach mathematics. Standardized data designate this as the period when females begin to underperform their male peers (NRC, 2013; U.S. Department of Education, Institute of Education Sciences, 2019). The findings support the development and early implementation of pedagogical models used to increase mathematical confidence in females.

Satisfaction

It was no surprise that females in the experiential learning course reported a significantly higher level of satisfaction associated with learning geometry. The final project, a small house that was donated to a charity serving the needs of homeless members of the community, may have provided a sense of satisfaction that their efforts to learn geometry were worthwhile and connected to a humanitarian effort (Taketa, 2017). This is consistent with the natural gratification that occurs when students are provided meaningful opportunities to utilize the knowledge and skills acquired during instruction (Keller, 1987a).

The empirical evidence collected suggests that conditions in the experiential GIC course positively affected male and female motivation to learn geometry. Therefore, educators should consider developing similar experiential, service learning models appropriate for other mathematics strands and STEM disciplines. Innovative STEM curricula and transformative instructional models could be used to motivate more secondary students to pursue STEM careers. This could be especially pertinent for motivating females as they are one of several minority groups severely underrepresented in STEM.

Conclusion 2: Females who completed the year-long experiential geometry curriculum demonstrated higher achievement in geometry than females who completed the traditional curriculum.

Conclusion 3: There were no significant differences in achievement in geometry amongst males and females who experienced the year-long experiential geometry curriculum.

Conclusions two and three, when viewed together, address the literature claiming that a widening achievement gap in mathematics exists between males and females at the secondary level. The empirical evidence collected on the achievement in geometry of males and females completing an experiential curriculum refutes the claim made by Ganley & Lubienski (2016) that secondary males outperform their female peers in advanced concepts such as problem solving and spatial reasoning. Females in the GIC course demonstrated achievement in geometry equivalent to their male peers who simultaneously experienced the course. In addition, females in the experiential GIC course showed higher achievement in geometry than males and females who experienced the traditional geometry course (see Figure 11). This finding is supported by several

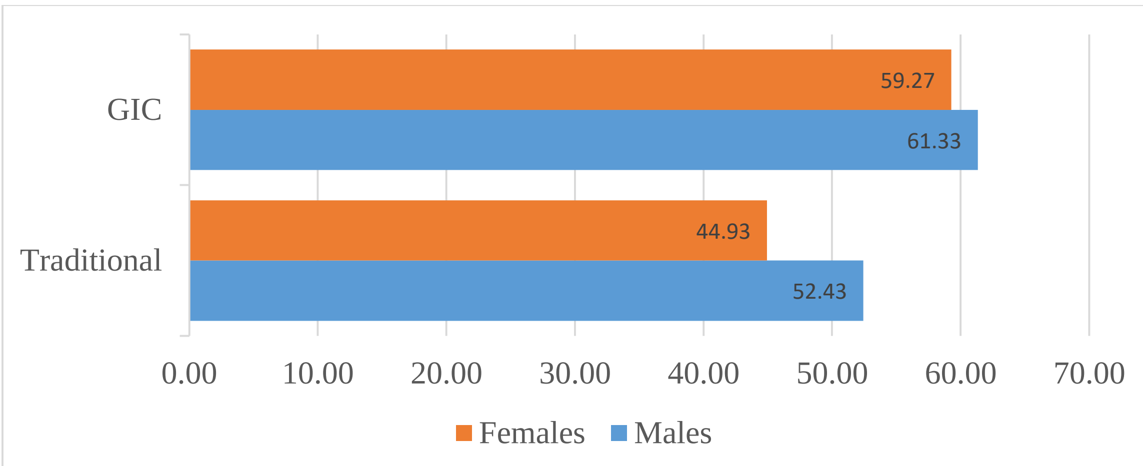

Figure 11. Achievement scores in geometry: Comparison by gender and group.

studies that argued there are no inherent genetic or biological differences between the male and female capacity for learning mathematics (Ganley & Lubienski, 2016; Lindberg, Hyde, Peterson, & Linn, 2010; Andreescu et al. 2008). These studies identified environmental and contextual factors such as confidence, interest and achievement to be the main cause of gender-based differences in mathematics performance (Ganely & Lubienski, 2016).

Students taking the experiential learning course displayed higher achievement in geometry (see Figure 12) which may suggest an overall improvement in their ability to reason, make accurate computations, and solve problems. The National Research Council (2001)

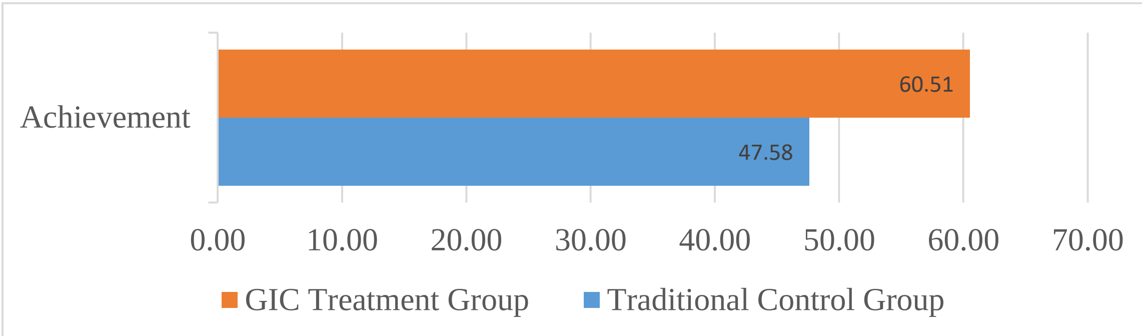

Figure 12. Achievement scores in geometry: Comparison by group.

described mathematical proficiency as successful development in five strands: concept attainment, accurate computation, problem solving, reasoning, confidence and value for use. The NRC (2001) further acknowledged that the most effective learning environments incorporate varied instructional approaches in which multiple strands are addressed simultaneously. The GIC instructional model provided a context that supports the development of mathematical proficiency as described by the NRC (2001). Contrary to the traditional geometry course, where students were taught using a recitation model, the GIC instructional model incorporated a blend of teacher directed and student-centered instruction. The GIC course was co-taught by a career technical education and a general geometry education instructor. The students moved beyond learning concepts in isolation with the geometry instructor as they were provided first-hand experience practicing acquired learning with the career technical education instructor who facilitated the construction project. The process of designing and constructing the house created daily opportunities for the students to simultaneously engage in all five proficiency strands.

This approach aligns with the literature citing the effectiveness of balancing rote memorization and problem solving for developing deeper understanding and proficiency with mathematical concepts (Bruner, 1977; Larson & Kanold, 2016; NCTM, 2000; NMAP, 2008; NRC, 2001, Sherman et al., 2013).

The need to improve instructional practices in mathematics remains, but the results help bridge the gap between mathematics education research and classroom practices. In addition, the results illustrate an effective response to the call by PCAST (2012) to develop transformative instructional models that enhance interest and improve preparation for careers in STEM.

Recommendations for Practitioners

Based on the findings, the following recommendations are suggested for practitioners in secondary mathematics and STEM education:

1. In order to enhance motivation of STEM learners, instructors should utilize John Keller's (2010) ARCS motivational model as a framework for arranging resources procedures, and experiences.
2. Curriculum designers and mathematics instructors should enhance mathematics experiential learning models by including a service-learning component. Mathematics instruction emphasizing interactions with people and the benefits mathematics provides to humans and society may align better with the personal and occupational values of females and motivate more females to pursue careers in STEM (Eccles & Wang, 2016).

3. Curriculum designers and mathematics instructors should use the GIC model to develop learning environments that help students reduce anxiety and build confidence for learning mathematics. Ganley and Lubienski (2016) found female lack of confidence to be the largest gender-based gap associated with learning mathematics.
4. Curriculum designers and mathematics instructors should replicate components of the experiential, non-traditional GIC model in other mathematics strands. Additionally, school systems should provide professional development to assist mathematics teachers with strengthening their learning experiences so that they move beyond the recitation model that is commonly implemented. Recitation as a lone instructional approach rejects the learning progression theory of Bruner and the motivational design theory of Keller (Bruner, 1977; Grouws & Smith, 2000; Keller 1987a; Stigler et al., 1999).
5. Practitioners should intervene early and offer similar models for learning STEM subjects at the elementary and middle school level to ensure equivalent development of spatial reasoning among genders. Spatial reasoning, the ability to envision, orient, and manipulate objects in three-dimensional space, has been used as a predictor of mathematical achievement and future success in STEM for a long time (Lowrie & Jorgensen, 2018; Wai, Lubinski, & Benbow, 2009).
6. Universities should model the use of non-traditional, experiential mathematics curricula, like GIC, with pre-service mathematics teachers to expose them to

methodologies that converge classroom learning with real work application and 21st century workforce expectancies. Nadelson et al. (2013) noted that student achievement in STEM is often hampered by teachers' lack of confidence, constrained background, and efficacy for teaching STEM.

Recommendations for Future Research

Continued research toward improving mathematical instruction through the development and exploration of innovative pedagogical models is encouraged. Several questions arose that were beyond the focus of the four research questions framing this investigation.

1. What effects would a similar treatment produce in other STEM disciplines?
2. What effects would a similar treatment produce in other mathematics strands?
3. What effects does GIC have on males and females who are situated in an urban setting?
4. Does experiencing GIC affect the decision of females to participate in other mathematics coursework?
5. Does experiencing GIC affect the desire of students to pursue further STEM coursework and careers?
6. What effects would similar treatments produce at the elementary and middle school levels?
7. Are there differences in long term geometry concept attainment after experiencing the GIC course?

Concluding Remarks

Over the past decade, schools have begun to employ non-traditional, innovative curricula in mathematics and other STEM related courses. Some of the most salient experiences are those that allow for authentic classroom projects through partnerships with community professionals. When planned and purposefully implemented, such programs, including Geometry In Construction, augment motivation and achievement.

The overall results suggest an effective model that can be used to motivate secondary males and females to learn geometry. Geometry In Construction is one example of a non-traditional mathematics curricular model. It is aligned with mathematics education research that proposes how mathematics should be taught in the classroom in order for students to develop proficiency. The design and instructional practices used in this model were shown to not only impact achievement and motivation of male and female participants, but critically noted is the significant impact it had on female confidence toward learning geometry.

It is our hope that knowledge gained from the findings will help educators design and implement other STEM curricula that increases interest and better prepares students for success in STEM careers. Educational researchers and practitioners who respond to the PCAST (2012) call to develop transformative instructional models for STEM education will help ensure the U.S. remains a technological leader. It is our job as practitioners to inspire students to pursue fulfilling careers such as those offered in STEM fields. We must be intentional in our efforts to diversify STEM by enhancing the number of females and minorities pursuing and succeeding in STEM careers so that the world benefits from the talents of all individuals.

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APPENDIX A

Comparison of Missouri Learning Standards to Common Core Standards for Geometry

Missouri Learning Standards	Common Core Standards
Experiment with transformations in the plane. G.CO.A	***Congruence*** ***CCSS.MATH.CONTENT.HSG.CO.A***
G.CO.A.1 Define angle, circle, perpendicular line, parallel line, line segment and ray based on the undefined notions of point, line, distance along a line and distance around a circular arc.	**CCSS.MATH.CONTENT.HSG.CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
G.CO.A.2 Represent transformations in the plane, and describe them as functions that take points in the plane as inputs and give other points as outputs.	**CCSS.MATH.CONTENT.HSG.CO.A.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
G.CO.A.3 Describe the rotational symmetry and lines of symmetry of two dimensional figures.	**CCSS.MATH.CONTENT.HSG.CO.A.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
G.CO.A.4 Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.	**CCSS.MATH.CONTENT.HSG.CO.A.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.CO.A.5 Demonstrate the ability to rotate, reflect or translate a figure, and determine a possible sequence of transformations between two congruent figures.	**CCSS.MATH.CONTENT.HSG.CO.A.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
G.CO.B ***Understand congruence in terms of rigid motions.***	***CCSS.MATH.CONTENT.HSG.CO.B***
G.CO.B.6 Develop the definition of congruence in terms of rigid motions.	**CCSS.MATH.CONTENT.HSG.CO.B.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use

	the definition of congruence in terms of rigid motions to decide if they are congruent.
G.CO.B.7 Develop the criteria for triangle congruence from the definition of congruence in terms of rigid motions.	**CCSS.MATH.CONTENT.HSG.CO.B.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. **CCSS.MATH.CONTENT.HSG.CO.B.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
G.CO.C ***Prove geometric theorems.***	***CCSS.MATH.CONTENT.HSG.CO.C***
G.CO.C.8 Prove theorems about lines and angles.	**CCSS.MATH.CONTENT.HSG.CO.C.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
G.CO.C.9 Prove theorems about triangles.	**CCSS.MATH.CONTENT.HSG.CO.C.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
G.CO.C.10 Prove theorems about polygons.	**CCSS.MATH.CONTENT.HSG.CO.C.11** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
G.CO.D ***Make geometric constructions.***	**CCSS.MATH.CONTENT.HSG.CO.D**
G.CO.D.11 Construct geometric figures using various tools and methods.	**CCSS.MATH.CONTENT.HSG.CO.D.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

	CCSS.MATH.CONTENT.HSG.CO.D.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
G.SRT.A ***Understand similarity in terms of similarity transformations.*** **G.SRT.A.1** Construct and analyze scale changes of geometric figures. **G.SRT.A.2** Use the definition of similarity to decide if figures are similar and to solve problems involving similar figures. **G.SRT.A.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar	***Similarity, Right Triangles, & Trigonometry*** ***CCSS.MATH.CONTENT.HSG.SRT.A*** **CCSS.MATH.CONTENT.HSG.SRT.A.1** Verify experimentally the properties of dilations given by a center and a scale factor: **CCSS.MATH.CONTENT.HSG.SRT.A.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **CCSS.MATH.CONTENT.HSG.SRT.A.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.
G.SRT.B ***Prove theorems involving similarity.*** **G.SRT.B.4** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	***CCSS.MATH.CONTENT.HSG.SRT.B*** **CCSS.MATH.CONTENT.HSG.SRT.B.4** Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.* **CCSS.MATH.CONTENT.HSG.SRT.B.5** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.
G.SRT.C Define trigonometric ratios, and solve problems involving right triangles. **G.SRT.C.5** Understand that side ratios in right triangles define the trigonometric ratios for acute angles. **G.SRT.C.6** Explain and use the relationship between the sine and cosine of complementary angles.	***CCSS.MATH.CONTENT.HSG.SRT.C*** ***CCSS.MATH.CONTENT.HSG.SRT.D*** **CCSS.MATH.CONTENT.HSG.SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **CCSS.MATH.CONTENT.HSG.SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles.

G.SRT.C.7 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles. **G.SRT.C.8** Derive the formula A = 1/2 ab sin(C) for the area of a triangle.	**CCSS.MATH.CONTENT.HSG.SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **CCSS.MATH.CONTENT.HSG.SRT.D.9** Derive the formula *A* = 1/2 *ab* sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **CCSS.MATH.CONTENT.HSG.SRT.D.10** Prove the Laws of Sines and Cosines and use them to solve problems. **CCSS.MATH.CONTENT.HSG.SRT.D.11** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
G.C.A ***Understand and apply theorems about circles.*** **G.C.A.1** Prove that all circles are similar using similarity transformations. **G.C.A.2** Identify and describe relationships among inscribed angles, radii and chords of circles. **G.C.A.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	***Circles*** ***CCSS.MATH.CONTENT.HSG.C.A*** **CCSS.MATH.CONTENT.HSG.C.A.1** Prove that all circles are similar. **CCSS.MATH.CONTENT.HSG.C.A.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **CCSS.MATH.CONTENT.HSG.C.A.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **CCSS.MATH.CONTENT.HSG.C.A.4** Construct a tangent line from a point outside a given circle to the circle.
G.C.B ***Find arc lengths and areas of sectors of circles.*** **G.C.B.4** Derive the formula for the length of an arc of a circle. **G.C.B.5**	***CCSS.MATH.CONTENT.HSG.C.B*** **CCSS.MATH.CONTENT.HSG.C.B.5** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as

Derive the formula for the area of a sector of a circle.	the constant of proportionality; derive the formula for the area of a sector.
G.GPE.A ***Translate between the geometric description and the equation for a conic section.*** **G.GPE.A.1** Derive the equation of a circle. **G.GPE.A.2** Derive the equation of a parabola given a focus and directrix.	***Expressing Geometric Properties with Equations*** ***CCSS.MATH.CONTENT.HSG.GPE.A*** **CCSS.MATH.CONTENT.HSG.GPE.A.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **CCSS.MATH.CONTENT.HSG.GPE.A.2** Derive the equation of a parabola given a focus and directrix. **CCSS.MATH.CONTENT.HSG.GPE.A.3** Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
G.GPE.B ***Use coordinates to prove geometric theorems algebraically.*** **G.GPE.B.3** Use coordinates to prove geometric theorems algebraically. **G.GPE.B.4** Prove the slope criteria for parallel and perpendicular lines and use them to solve problems. **G.GPE.B.5** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G.GPE.B.6** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles.	***CCSS.MATH.CONTENT.HSG.GPE.B*** **CCSS.MATH.CONTENT.HSG.GPE.B.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **CCSS.MATH.CONTENT.HSG.GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **CCSS.MATH.CONTENT.HSG.GPE.B.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **CCSS.MATH.CONTENT.HSG.GPE.B.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

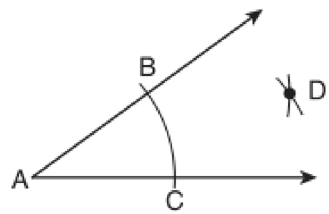
G.GMD.A ***Explain volume formulas and use them to solve problems*.** **G.GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid and cone. **G.GMD.A.2** Use volume formulas for cylinders, pyramids, cones, spheres and composite figures to solve problems.	***Geometric Measurement & Dimension*** ***CCSS.MATH.CONTENT.HSG.GMD.A*** **CCSS.MATH.CONTENT.HSG.GMD.A.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*. **CCSS.MATH.CONTENT.HSG.GMD.A.2** Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. **CCSS.MATH.CONTENT.HSG.GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
G.GMD.B ***Visualize relationships between two-dimensional and threedimensional objects.*** **G.GMD.B.3** Identify the shapes of two-dimensional cross-sections of three dimensional objects. **G.GMD.B.4** Identify three-dimensional objects generated by transformations of two-dimensional objects.	***CCSS.MATH.CONTENT.HSG.GMD.B*** **CCSS.MATH.CONTENT.HSG.GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
G.MG.A ***Apply geometric concepts in modeling situations.*** **G.MG.A.1** Use geometric shapes, their measures and their properties to describe objects. **G.MG.A.2** Apply concepts of density based on area and volume in modeling situations. **G.MG.A.3** Apply geometric methods to solve design mathematical modeling problems.	***Modeling with Geometry*** ***CCSS.MATH.CONTENT.HSG.MG.A*** **CCSS.MATH.CONTENT.HSG.MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).[*] **CCSS.MATH.CONTENT.HSG.MG.A.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).[*] **CCSS.MATH.CONTENT.HSG.MG.A.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).[*]

G.CP.A ***Understand independence and conditional probability and use them to interpret data.*** **G.CP.A.1** Describe events as subsets of a sample space using characteristics of the outcomes, or as unions, intersections or complements of other events. **G.CP.A.2** Understand the definition of independent events and use it to solve problems. **G.CP.A.3** Calculate conditional probabilities of events. **G.CP.A.4** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. **G.CP.A.5** Recognize and explain the concepts of conditional probability and independence in a context. **G.CP.A.6** Apply and interpret the Addition Rule for calculating probabilities. **G.CP.A.7** Apply and Interpret the general Multiplication Rule in a uniform probability model. **G.CP.A.8** Use permutations and combinations to solve problems.	

APPENDIX B

2018 Missouri Geometry End of Course Practice Exam

1. A student is working on a geometric construction.



If $\overrightarrow{AD}$ is drawn, what geometric construction is shown?

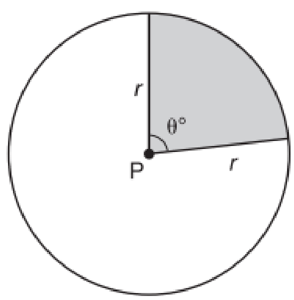
A. angle bisector

B. copying an angle

C. perpendicular bisector

D. measuring an angle

2. A sector is part of a circle's area that is defined by a central angle. The ratio of the sector's area, A, to the circle's area, πr^2, is identical to the ratio of the central angle, θ, to the total measure of the circle, 360°.



Which option represents the formula for the area of a sector?

A. $A = \frac{360°}{\theta} \pi r^2$

B. $A = \theta\pi r^2$

C. $A = 360°\pi r^2$

D. $A = \frac{\theta}{360°} \pi r^2$

3. Select the responses that correctly complete the sentence.

   Given △ABC with A(5, 4), B(2, −2), and C(7, −1).

   △ABC is classified as __________ because ______________________ .

   ○ scalene ○ all three sides are congruent
   ○ isosceles ○ two of the sides are congruent
   ○ equilateral ○ no two sides are congruent

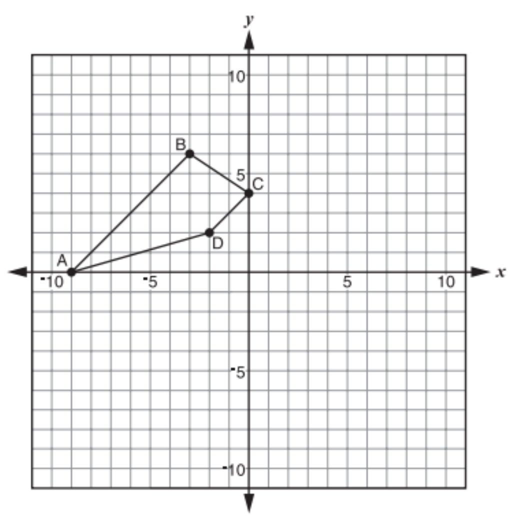
4. Point P is between D(2, 5) and F(5, −1). What are the coordinates of P along the directed $\overline{DF}$ if the ratio of DP to PF is 1:2?

   Enter the correct coordinates in the boxes.

   Value of the x-coordinate: ☐ Value of the y-coordinate: ☐

5. Given quadrilateral ABCD, what are the coordinates for the resulting image, A″B″C″D″, after the two transformations listed?

   First transformation: Rotate 90° clockwise about the origin.
   Second transformation: Translate (x + 1, y − 2).



   Enter the coordinates for the resulting image A″B″C″D″ in the boxes.

   A″ = (☐ , ☐) C″ = (☐ , ☐)

   B″ = (☐ , ☐) D″ = (☐ , ☐)

6. The following table provides a list of four international cities, their populations, and the area of the cities.

City/Urban Area	Country	Population (in million people)	Area (square miles)
Tokyo	Japan	33.2	2,697
Sao Paulo	Brazil	17.7	759
Seoul	Korea	17.5	405
Mexico City	Mexico	17.4	799

Determine the population densities of each city and then order them from least to greatest. Which list shows the population densities of each city in order from least to greatest?

A. Mexico City, Sao Paulo, Seoul, Tokyo

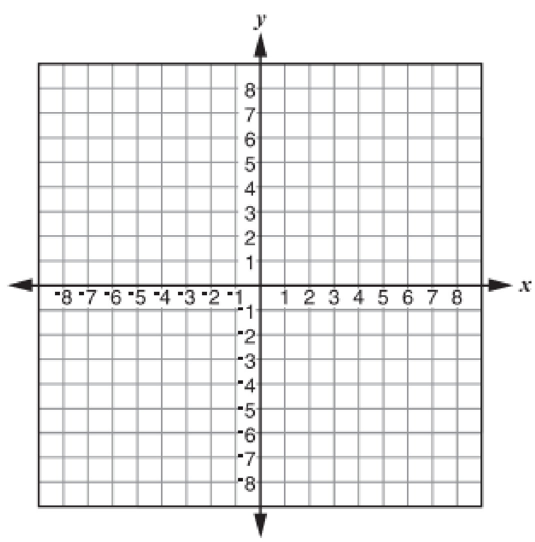
B. Sao Paulo, Tokyo, Seoul, Mexico City

C. Seoul, Sao Paulo, Mexico City, Tokyo

D. Tokyo, Mexico City, Sao Paulo, Seoul

7. The endpoints of $\overline{AB}$ are A(1, 2) and B(5, 6). Line k is the perpendicular bisector of $\overline{AB}$.

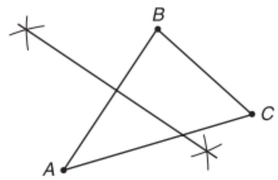
Graph $\overline{AB}$ and line k.



8. What are the possible cross sections of a right circular cone?
Select all that apply.

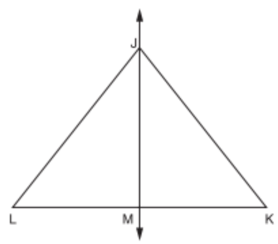
A. ellipse
B. triangle
C. circle
D. parabola
E. rectangle

9. Olivia is constructing the circumscribed circle of a triangle as shown in the diagram. What should be her next step in the process?



A. Construct the angle bisector of ∠A.
B. Construct the perpendicular bisector of $\overline{BC}$.
C. Set the compass width to AB, then draw a circle with center point A.
D. Set the compass width to BC, then draw a circle with center point C.

10. Given: $\overleftrightarrow{JM}$ is the perpendicular bisector of $\overline{LK}$
Prove: J is equidistant from L and K D. E. F.



Mark the letters in the table for the statements that complete the proof correctly.

Statements	Reasons
1. $\overline{JM}$ is the perpendicular bisector of $\overline{LK}$	1. Given
2. ∠LMJ and ∠JMK are right angles	2. A B C D E F
3. A B C D E F	3. All right angles are congruent
4. $\overline{LM} \cong \overline{MK}$	4. Definition of bisector
5. A B C D E F	5. Reflexive property of equality
6. ΔLMJ ≅ ΔKMJ	6. SAS
7. $\overline{LJ} \cong \overline{JK}$	7. A B C D E F
8. J is equidistant from L and K	8. Definition of equidistant

A. ∠LMJ ≅ ∠JMK

B. Definition of right angle

C. Corresponding parts of congruent triangles are congruent.

D. Definition of perpendicular

E. ∠LJM ≅ ∠KJM

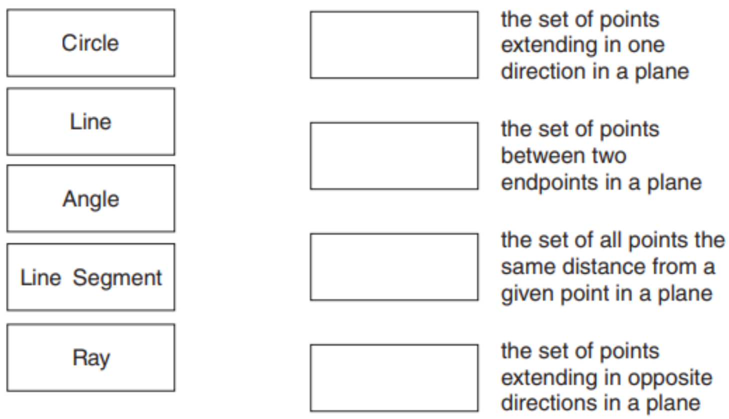
F. $\overline{JM} \cong \overline{JM}$

11. A circle has its center at (−2, 3) and point (4, 6) is on its circumference. What is the correct written equation of the circle?

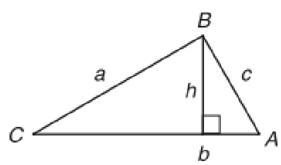
A. $(x + 3)^2 + (y - 2)^2 = 85$ C. $(x - 2)^2 + (y + 3)^2 = 85$

B. $(x - 3)^2 + (y + 2)^2 = 45$ D. $(x + 2)^2 + (y - 3)^2 = 45$

12. Draw a line from the words to the correct descriptions. Not all options will be used.



13. Triangle ABC is shown. The lengths of the sides of the triangle are represented by a, b, and c.



Select the next step that is needed to derive the equation for the area of triangle ABC when sides BC, AC, and the included angle C are given.

Step 1: $A = \frac{1}{2}bh$

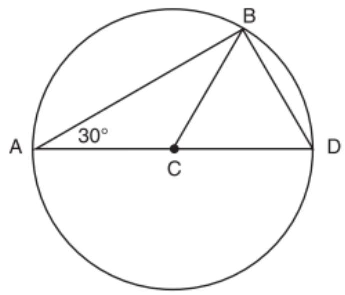
Step 2: $\sin C = \frac{h}{a}$

Step 3: __________

- O $h = (2A)/b$
- O $a = h \cdot \sin C$
- O $\cos C = a/h$
- O $h = a \cdot \sin C$

Step 4: $A = \frac{1}{2}ab \cdot \sin C$

14. Points A, B, and D lie on circle C.

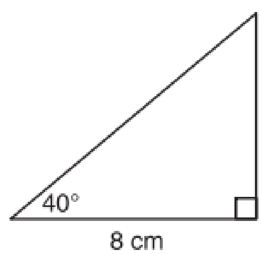


Determine the measure of the indicated angles given that $m\angle A = 30°$.
Enter the measures in the boxes.

$m\angle BCD =$ $\boxed{}$ °

$m\angle ABD =$ $\boxed{}$ °

15. The right triangle shown is missing the lengths of two sides.



Enter the lengths of the two missing sides in the boxes below. Round your answers to the nearest tenth.

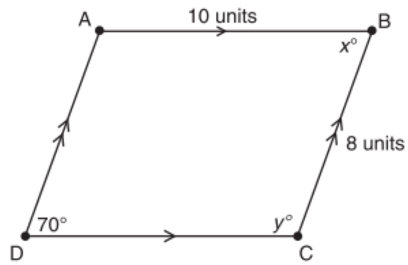
Length of the hypotenuse: $\boxed{}$ cm

Length of the leg: $\boxed{}$ cm

16. Which of the following two-dimensional cross sections are circles? Select all that apply.

A. any cross section of a sphere
B. horizontal cross section of a cube
C. cross section of a cone parallel to its base
D. cross section of a cone perpendicular to its base
E. cross section of a right cylinder parallel to its base
F. cross section of a pyramid perpendicular to its base

17. Parallelogram ABCD is shown.

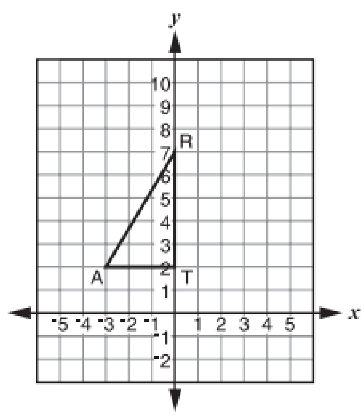


What are the values of x and y?

the correct values in the boxes.

x = ☐ y = ☐

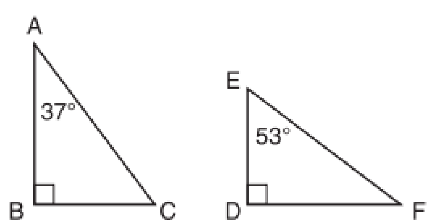
18. On the coordinate plane, △ART is shown with points R and T plotted on the y-axis.



What three-dimensional figure is created by rotating △ART around the y-axis?

A. cone C. cylinder
B. sphere D. pyramid

19. Two triangles are shown.



Which is a true statement about the two triangles?

A. The triangles are not similar.
B. △ABC ~ △EDF by AA Similarity Postulate
C. △ABC ~ △FDE by SAS Similarity Postulate
D. △ABC ~ △FDE by AA Similarity Postulate

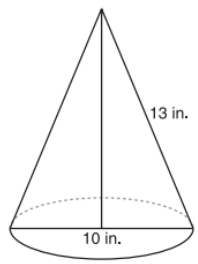
20. Select the values that correctly complete the sentence about the symmetry of a regular pentagon.

A regular pentagon has _____ lines of symmetry and has ___________ rotational symmetry.

- O 1
- O 2
- O 5
- O 6

- O 60 degree
- O 72 degree
- O 108 degree
- O 540 degree

21. A right cone has a diameter of 10 inches and a slant height of 13 inches. The cone is shown.



Which is the volume of the cone?

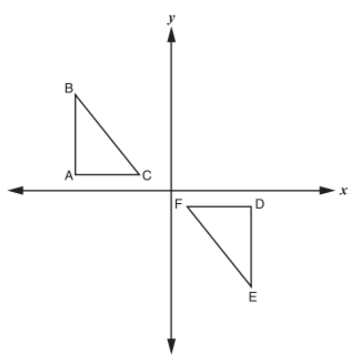
A. 100π in.^{3}

B. 400π in.^{3}

C. $\frac{325}{3}\pi$ in.^{3}

D. $\frac{1300}{3}\pi$ in.^{3}

22. △ABC and △DEF are plotted on the coordinate plane shown.



Which conclusions can be made about △ABC and △DEF if △ABC is mapped onto △DEF by reflecting △ABC over the y-axis and reflecting it over the x-axis? Select all that apply.

A. △ABC ≅ △DEF

B. The corresponding sides are proportional: $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$.

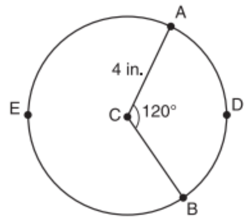
C. Reflecting △ABC across the *y*-axis and then the *x*-axis yields the same transformation as rotating △ABC 90° counterclockwise around the origin.

D. △ABC ~ △DEF

E. △ABC is acute, but △DEF is obtuse.

F. All corresponding sides are congruent.

23. Points A, B, D, and E lie on circle C.



What is the length of $\widehat{ADB}$?

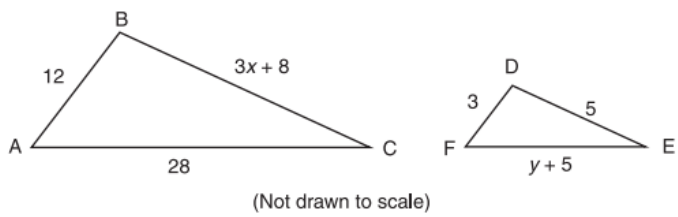
A. $\frac{8\pi}{3}$ in.

B. 4π in.

C. $\frac{16\pi}{3}$ in.

D. 8π in.

24. Given $\triangle ABC \sim \triangle FDE$, what are the values of x and y?



Select all that apply.

A. x = −1	D. y = −2
B. x = 2	E. y = 2
C. x = 4	F. y = 23

25. A candle maker has 301.59 cubic centimeters (cm3) of liquid wax to make cone-shaped candles. Each candle has a circular base with a diameter of 3 cm and a height of 5 cm. What is the maximum number of candles that can be made from the liquid wax?

A. 6	C. 25
B. 7	D. 26

26. Which is the equation of the parabola with focus (2, 5) and directrix y = 3?

A. $y = -\frac{1}{2}x^2 - x + \frac{5}{2}$

B. $y = -x^2 + 5x - 20$

C. $y = \frac{1}{4}x^2 - x + 5$

D. $y = \frac{1}{2}x^2 + x + 6$

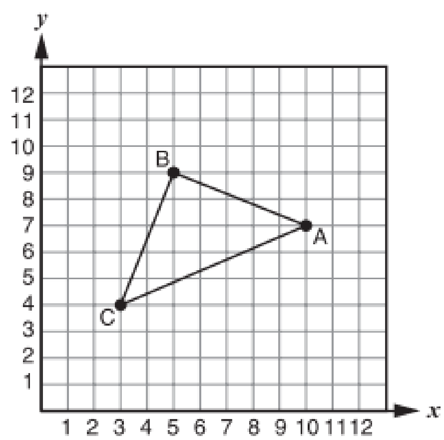
27. Britney found an irregularly shaped metal object on the beach that has a mass of 232.5 grams. To determine the volume, she partially filled a cylindrical water bottle and dropped the object in. The water level in the bottle rose by 1.2 cm. The bottle has a diameter of 5 cm. Calculate the density of the metal to determine what type of metal Britney found. Densities, measured in grams per cubic centimeter, $\frac{g}{cm^3}$, for some common metals are listed.

• Copper: 8.86 $\frac{g}{cm^3}$ • Bronze: 9.87 $\frac{g}{cm^3}$ • Silver: 10.5 $\frac{g}{cm^3}$ • Gold: 19.3 $\frac{g}{cm^3}$

Select the word that correctly completes the sentence. Based on the density of the metal, it is most likely that the metal Britney found is _______________.

a. copper c. silver
b. bronze d. gold

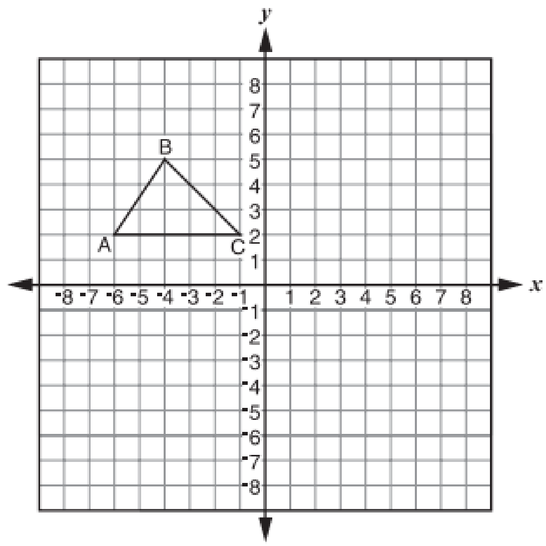
28. In △ABC, ∠B is a right angle. The coordinates for each point are A(10, 7), B(5, 9), and C(3, 4).



Rounded to the nearest tenth, what is the area, in square units, of △ABC? Enter the area in the box.

☐ $units^2$

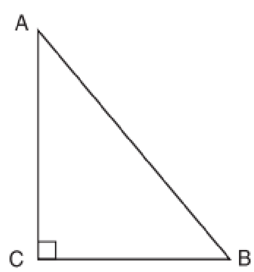
29. △ABC is shown on the coordinate plane.



After rotating △ABC 180° about the origin and then reflecting it over the x-axis, what are the coordinates of △A″B″C″?

A. A″(2, 6), B″(5, 4), C″(2, 1)
B. A″(6, 2), B″(4, 5), C″(1, 2)
C. A″(−2, 6), B″(5, 4), C″(−2, 1)
D. A″(6, −2), B″(4, −5), C″(1, −2)

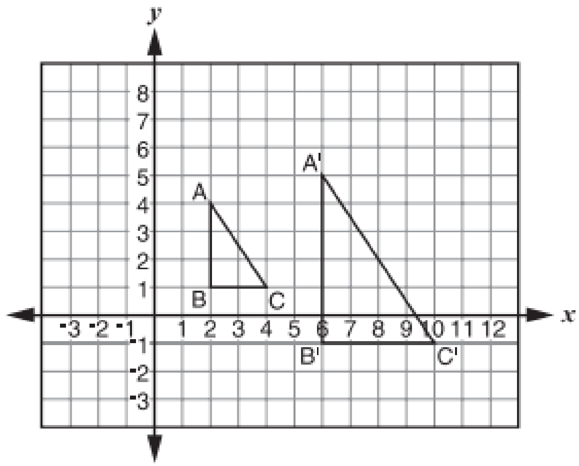
30. Right triangle ABC is shown.



What must be true about ∠A and ∠B? Select all that apply.

A. ∠A ≅ ∠B
B. ∠A and ∠B are complementary
C. ∠A and ∠B are supplementary
D. cos A = cos B
E. cos A = sin B
F. sin A = cos B
G. sin A = sin B

31. The pre-image of △ABC and its image △A'B'C' are shown on the coordinate plane.



Which rule describes the transformation represented in the graph?

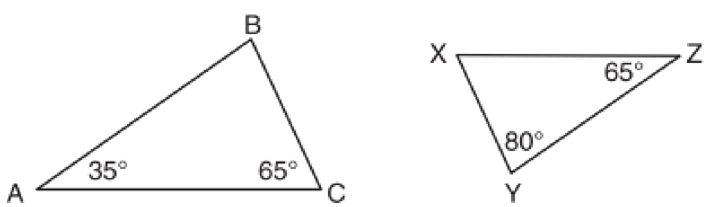
A. $\left(\frac{1}{2}x + 2, \frac{1}{2}y - 3\right)$

B. $(2x + 2, 2y - 3)$

C. $\left(\frac{1}{2}x - 2, \frac{1}{2}y + 3\right)$

D. $(2x - 2, 2y + 3)$

32. Two angle measures for both △ABC and △XYZ are given.



Using the given information about the triangles, is △ABC ~ △XYZ?

A. Yes, the triangles are similar by AA.
B. No, because only 1 pair of corresponding angles are congruent.
C. No, we cannot determine similarity without knowing the third angles.
D. No, we cannot determine similarity without knowing the side ratios.

APPENDIX C

John Keller's (2010) Course Interest Survey

Keller's Original Version of CIS

Student Instructions:

1. There are 34 statements in this section. Please think about each statement in relation to the instructional materials you have just studied, and indicate how true it is. Give the answer that truly applies to you, and not what you would like to be true, or what you think others want to hear.

2. Think about each statement by itself and indicate how true it is. Do not be influenced by your answers to other statements.

3. Record your responses on the answer sheet that is provided, and follow any additional instructions that may be provided in regard to the answer sheet that is being used with this survey. Thank you.

1 = Not true 2 = Slightly true 3 = Moderately true 4 = Mostly true 5 = Very true

1. The instructor knows how to make us feel enthusiastic about the subject matter of this course.
2. The things I am learning in this course will be useful to me.
3. I feel confident that I will do well in this course.
4. This class has very little in it that captures my attention.
5. The instructor makes the subject matter of this course seem important.
6. You have to be lucky to get good grades in this course.
7. I have to work too hard to succeed in this course.
8. I do NOT see how the content of my this course relates to anything I already know.
9. Whether or not I succeed in this course is up to me.
10. The instructor creates suspense when building up to a point.

11. The subject matter of this course is just too difficult for me.
12. I feel that this course gives me a lot of satisfaction.
13. In this class, I try to set and achieve high standards of excellence.
14. I feel the grades or other recognition I receive are fair compared to other students.
15. The students in this class seem curious about the subject matter.
16. I enjoy working for this course.
17. It is difficult to predict what grade the instructor will give my assignments
18. I am pleased with the instructor’s evaluations of my work compared to how well I think I have done.
19. I feel satisfied with what I am learning from this course.
20. The content of this course relates to my expectations and goals.
21. My instructor does unusual or surprising things that are interesting.
22. The students actively participate in this class.
23. To accomplish my goals, it is important that I do well in this course.
24. My instructor uses an interesting variety of teaching techniques.
25. I do NOT think I will benefit much from this course.
26. I often daydream while in this class.
27. As I am taking this class, I believe that I can succeed if I try hard enough.
28. The personal benefits of this course are clear to me.
29. My curiosity is often stimulated by the questions asked or the problems given on the subject matter in this class.
30. I find the challenge level in this course to be about right: neither too easy nor too hard.

31. I feel rather disappointed with this course.

32. I feel that I get enough recognition of my work in this course by means of grades, comments, or other feedback.

33. The amount of work I have to do is appropriate for this type of course.

34. I get enough feedback to know how well I am doing.

Customized Version of CIS

Student Instructions: There are 34 statements in this section. Please think about each statement in relation to the geometry class you are taking and indicate how true it is. Give the answer that truly applies to you, and not what you would like to be true, or what you think others want to hear.

Think about each statement by itself and indicate how true it is. Do not be influenced by your answers to other statements.

Click on the circle next to the response that best fits your experience so far. Use the following values to indicate your response to each item.

1 = Not true 2 = Slightly true 3 = Moderately true 4 = Mostly true 5 = Very true

1. My math teacher knows how to make us feel enthusiastic about math.
2. The things I am learning in math class will be useful to me.
3. I feel confident that I will do well in math class.
4. Math class has very little in it that captures my attention.
5. My teacher makes math seem important.
6. You have to be lucky to get good grades in my math class.
7. I have to work too hard to succeed in math class.
8. I do NOT see how the content of my math class relates to anything I already know.
9. Whether or not I succeed in math class is up to me.

10. My math teacher creates excitement when building up to a point.
11. The subject matter of my math class is just too difficult for me.
12. I feel that my math class gives me a lot of satisfaction.
13. In my math class, I try to set and achieve high standards of excellence.
14. I feel the grades or other recognition I receive in math are fair compared to other students.
15. The students in my math class seem curious about the subject matter.
16. I enjoy working in my math class.
17. It is difficult to predict what grade my teacher will give my math assignments.
18. I am pleased with my math teacher's evaluations of my work compared to how well I think I have done.
19. I feel satisfied with what I am learning from my math class.
20. The things I learn in math class meets my expectations and goals.
21. My math teacher does unusual or surprising things that are interesting.
22. The students actively participate in my math class.
23. To accomplish my goals, it is important that I do well in math class.
24. My math teacher uses a variety of teaching techniques.
25. I do NOT think I will benefit much from my math class.
26. I often daydream while in math class.
27. As I am taking this math class, I believe that I can succeed if I try hard enough.
28. The personal benefits of math class are clear to me.
29. My curiosity is often stimulated by the questions asked or the problems given in math class.

30. I find the challenge level in math class to be about right: neither too easy nor too hard.
31. I feel disappointed with math class.
32. I feel that I get enough recognition of my work in math class by means of grades, comments, or other feedback.
33. The amount of work I have to do is appropriate for this type of math class.
34. I get enough feedback to know how well I am doing in math class.

APPENDIX D

Statistical Analyses of Assumptions Passed for ANCOVA

In order to address research questions one and two, analysis of covariance (ANCOVA) was used to statistically analyze differences in geometry end of course exam scores between the treatment and control group while controlling for algebra I end of course exam scores. Laerd Statistics (2018c), described nine assumptions data must pass in order for ANCOVA to yield valid results. Following is an analysis of the data and evidence addressing those assumptions.

Assumption #1: The dependent variable and covariate are measured on a continuous scale.

The dependent variable, achievement in geometry, and the covariate, achievement in algebra I were continuous variables representing the percentage of correct test questions. The values could have been any number from 0 to 100.

Assumption #2: The independent variable consists of two categorical, independent groups.

The independent variable, group, consisted of two categories. Participants either experienced the Geometry In Construction curriculum (treatment group), or a traditional geometry curriculum (control group), but not both. Gender is a more complex variable including more than two categories, and participants were given a choice to self-report gender other than male or female. All participants reported a gender category of either male or female, therefore, the independent variable, gender, consisted of two categories.

Assumption #3: Groups are independent.

Each participant experienced only one geometry curriculum. No participant experienced both curricula nor did they identify with more than one gender, therefore there was no relationship between the data collected from participants within or between groups.

Assumption #4: There are no significant outliers.

Boxplots of algebra I and geometry end of course exam scores, separated by group and gender, and using 95% confidence limits, revealed only one significant outlier in the sample. The outlier was confirmed as an accurate data point and was not excluded from the data analysis.

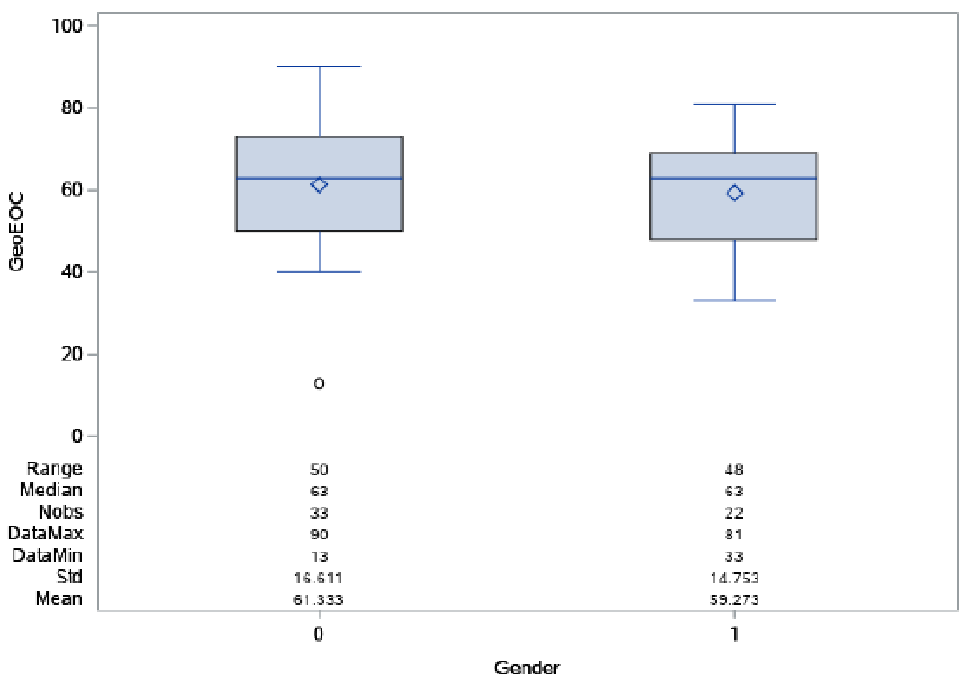


Figure D1. Distribution of treatment group geometry EOC exam scores by gender. 0 = male, 1 = female. One outlier identified among males.

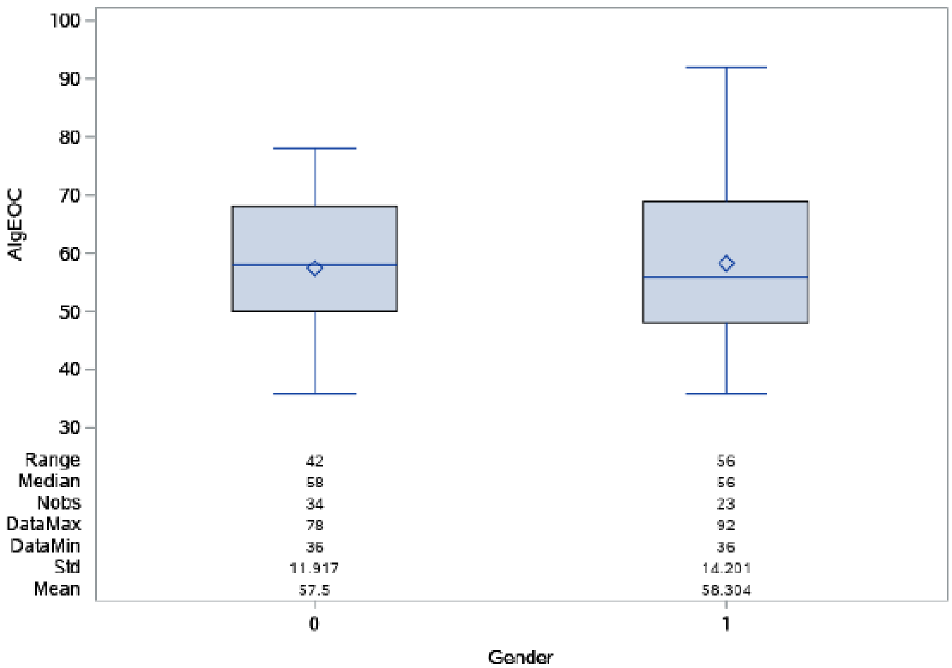


Figure D2. Distribution of treatment group algebra I EOC exam scores by gender.
0 = male, 1 = female.

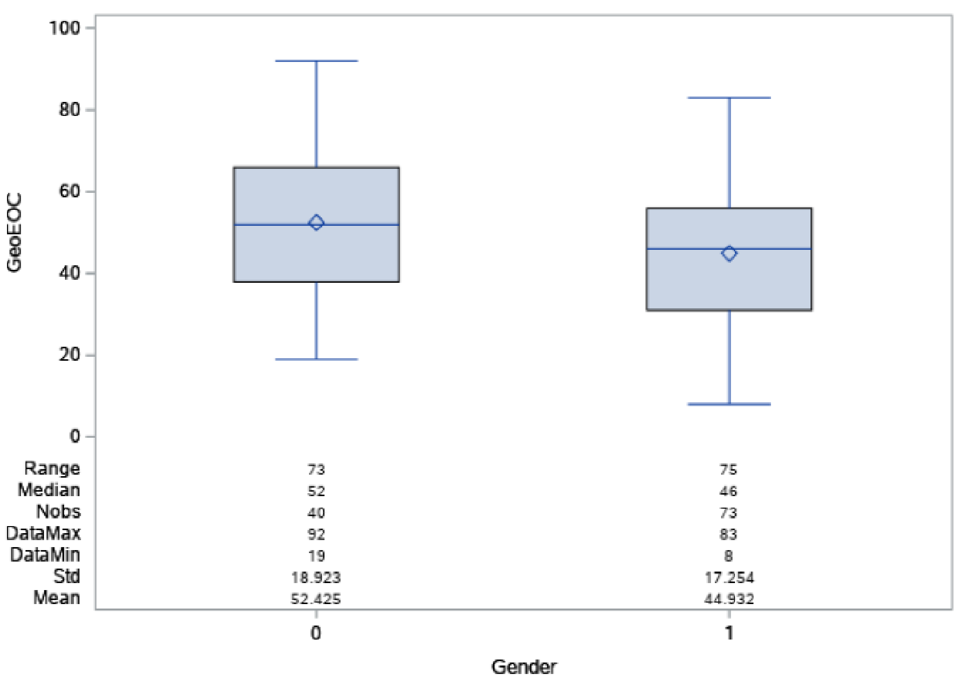


Figure D3. Distribution of control group geometry EOC exam scores by gender.
0 = male, 1 = female.

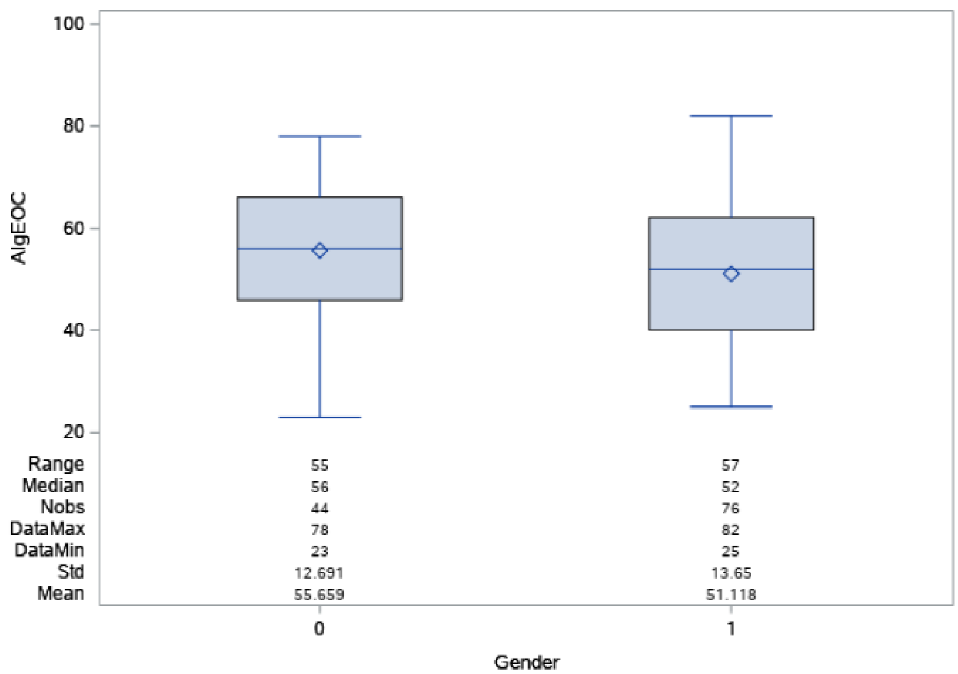


Figure D4. Distribution of control group algebra I EOC exam scores by gender. 0 = male, 1 = female.

Assumption #5: Residuals should be approximately normally distributed.

Assumption #6: Residuals demonstrate homoscedasticity.

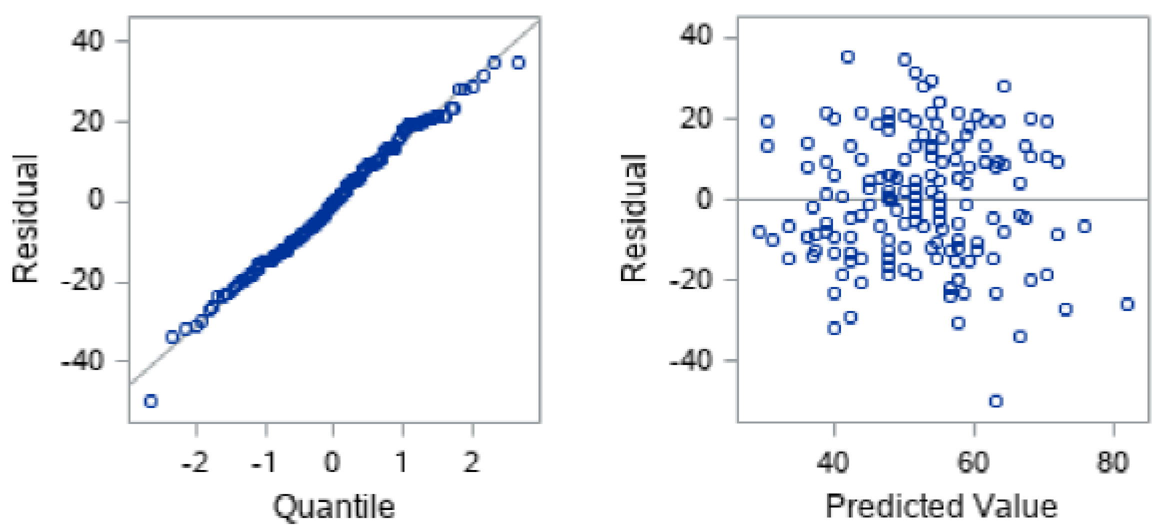


Figure D5. Analysis of residuals for all participants. Q-Q plot indicates residuals normally distributed. Scatter plot random pattern demonstrates homoscedasticity.

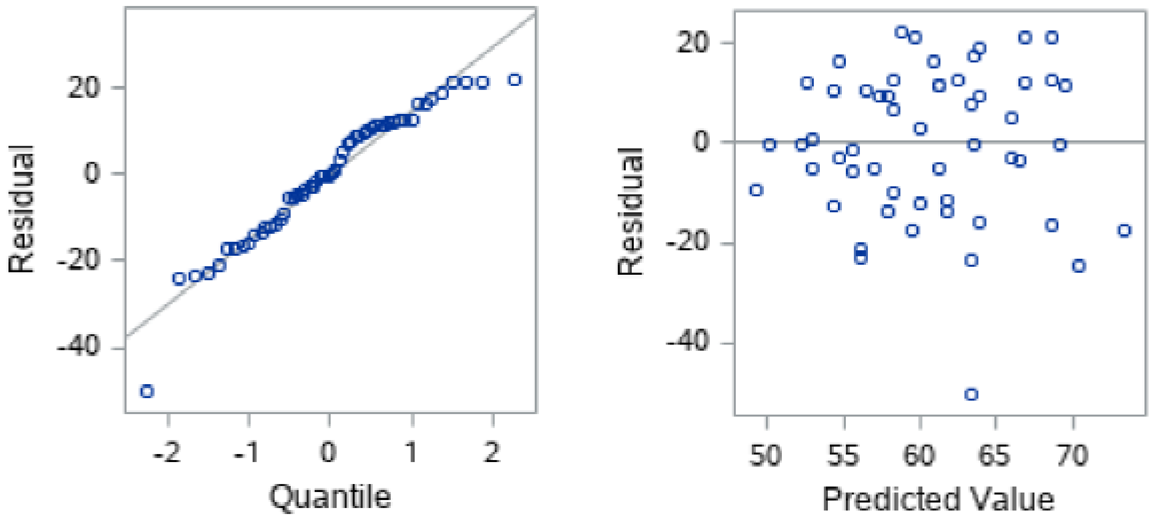


Figure D6. Analysis of residuals for treatment group. Q-Q plot indicates residuals nearly normally distributed. ANCOVA is robust to slight violations of normality. Scatter plot random pattern demonstrates homoscedasticity.

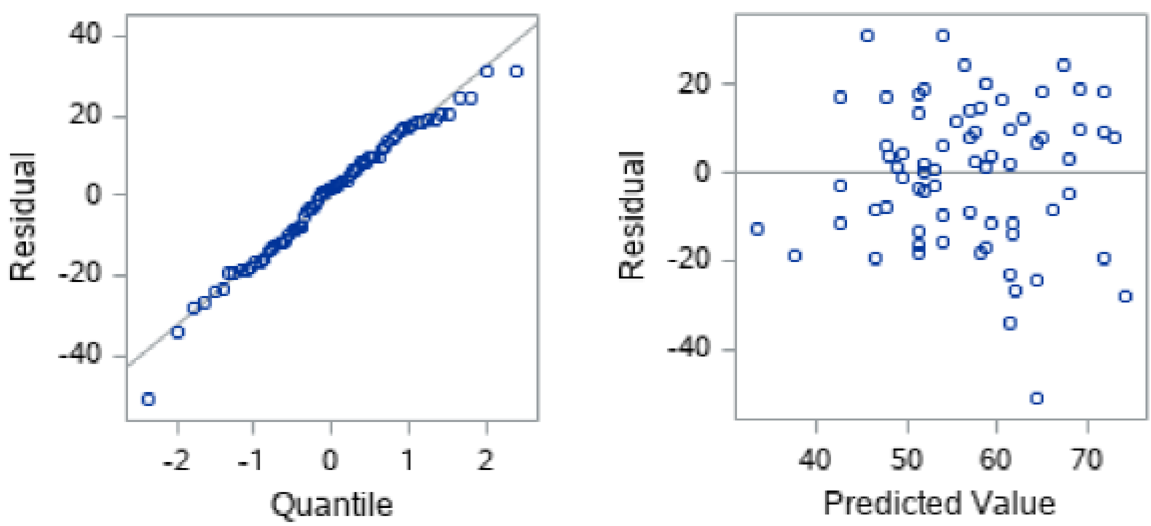


Figure D7. Analysis of residuals for males. Q-Q plot indicates residuals normally distributed. Scatter plot random pattern demonstrates homoscedasticity.

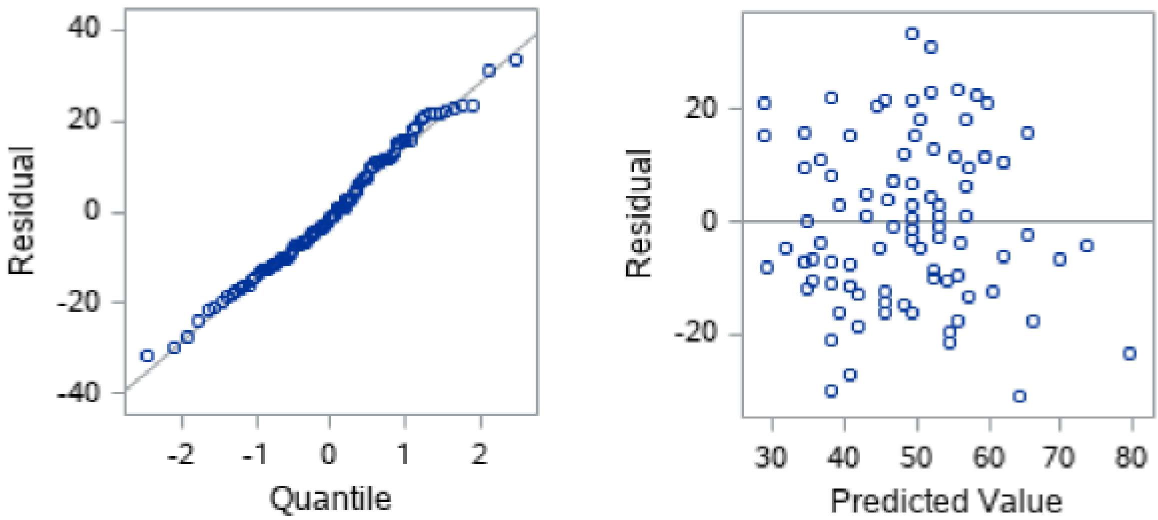


Figure D8. Analysis of residuals for females. Q-Q plot indicates residuals normally distributed. Scatter plot random pattern demonstrates homoscedasticity.

Assumption #7: Variances are equal between groups.

A folded F test was used to test for equality of variance in algebra I EOC exam scores and geometry EOC exam scores between groups.

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	119	56	1.11	0.6816

Figure D9. Analysis of variances in algebra I EOC exam scores between treatment and control groups. p value = 0.6816 indicates variances are equal.

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	112	54	1.32	0.2564

Figure D10. Analysis of variances in geometry EOC exam scores between treatment and control groups. p value = 0.2564 indicates variances are equal.

Assumption #8: The covariate is linearly related to the dependent variable.

Simple linear regression of geometry EOC exam scores on algebra I EOC exam scores was performed to determine the relationship between the covariate and the dependent variable.

Least Squares Model (No Selection)

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	1	13917	13917	54.63	<.0001
Error	162	41273	254.76865		
Corrected Total	163	55190			

Ho = Relationship between AlgEOC and GeoEOC scores described by this model is not significant.
If p<.05, reject null.
Since p=.0001, reject null.
Conclusion: There IS a significant linear relationship between AlgEOC and GeoEOC scores.

Root MSE	15.96147
Dependent Mean	52.09146
R-Square	0.2522
Adj R-Sq	0.2476
AIC	1076.60607
AICC	1076.75607
SBC	916.80580

About 25% of the variability in the GeoEOC scores are explained by the AlgEOC scores.

Parameter Estimates					
Parameter	**DF**	**Estimate**	**Standard Error**	**t Value**	**Pr > |t|**
Intercept	1	13.922890	5.312495	2.62	0.0096
AlgEOC	1	0.701439	0.094905	7.39	<.0001

p<.05 indicates the intercept and slope are significantly different from zero

y intercept

slope

Figure D11. Least squares model analysis of linear regression of dependent variable on covariate. Results indicate a significant linear relationship between dependent variable and covariate

Assumption #9: Regression slopes are equal.

An ***F*** test performed on a generalized linear model was used to identify interactions between independent variables and the covariate. When an ***F*** test is significant ($p < .05$), regression slopes are not equal.

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Course	1	160	5.22	0.0236
AlgEOC	1	160	33.17	<.0001
AlgEOC*Course	1	160	2.40	0.1230

Ho = Regression slopes for GIC and TG are equal.
If p<.05, reject null.
Since p=.1230, fail to reject null.
Conclusion: Regression slopes for GIC and TG are equal.

Figure D12. Comparison of the regression line slopes, when regressing geometry EOC exam scores on algebra I EOC exam scores, for treatment and control groups. Result of ***F*** test indicates the regression line slopes are equal.

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Gender	1	50	0.02	0.8846
AlgEOC	1	50	6.56	0.0135
AlgEOC*Gender	1	50	0.09	0.7644

Ho = Regression slopes for GIC males and females are equal.
If p<.05, reject null.
Since p=.7644, fail to reject null.
Conclusion: Regression slopes for GIC males and females are equal.

Figure D13. Comparison of the regression line slopes, when regressing geometry EOC exam scores on algebra I EOC exam scores, for males and females in the treatment group. Result of ***F*** test indicates the regression line slopes are equal.

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Course	1	67	1.24	0.2689
AlgEOC	1	67	13.49	0.0005
AlgEOC*Course	1	67	0.61	0.4389

Ho = Regression slopes for GIC males and TG males are equal.
If p<.05, reject null.
Since p=.4389, fail to reject null.
Conclusion: Regression slopes for GIC males and TG males are equal.

Figure D14. Comparison of the regression line slopes, when regressing geometry EOC exam scores on algebra I EOC exam scores, for males in the treatment and control groups. Result of ***F*** test indicates the regression line slopes are equal.

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Course	1	89	3.20	0.0771
AlgEOC	1	89	16.74	<.0001
AlgEOC*Course	1	89	1.53	0.2199

Ho = Regression slopes for GIC females and TG females are equal.
If p<.05, reject null.
Since p=.2199, fail to reject null.
Conclusion: Regression slopes for GIC females and TG females are equal.

Figure D15. Comparison of the regression line slopes, when regressing geometry EOC exam scores on algebra I EOC exam scores, for females in the treatment and control groups. Result of ***F*** test indicates the regression line slopes are equal.

APPENDIX E

Statistical Analyses of Assumptions Passed for t-tests

In order to address research questions three and four, independent t-tests were used to statistically analyze differences in scores between the treatment and control groups on a survey measuring motivation to learn geometry. Laerd Statistics (2018b), described six assumptions data must pass in order for t-tests to yield valid results. Following is an analysis of the data and evidence addressing those assumptions.

Assumption #1: The dependent variable is measured on a continuous scale.

There were five dependent variables related to motivation: overall motivation, attention, relevance, confidence, and satisfaction. Each dependent variable was measured on a whole number Likert Scale with values ranging from one to five.

Assumption #2: The independent variable consists of two categorical, independent groups.

The independent variable, group, consisted of two categories. Participants either experienced the Geometry In Construction curriculum (treatment group), or a traditional geometry curriculum (control group), but not both. Gender is a more complex variable including more than two categories, and participants were given a choice to self-report gender other than male or female. All participants reported a gender category of either male or female, therefore, the independent variable, gender, consisted of two categories.

Assumption #3: Groups are independent.

Each participant experienced only one geometry curriculum. No participant experienced both curricula nor did they identify with more than one gender, therefore there was no relationship between the data collected from participants within or between groups.

Assumption #4: There are no significant outliers.

Boxplots of motivation scores, separated by group and gender, and using 95% confidence limits, were used to identify outliers. The few outliers identified were confirmed as accurate data points and were not excluded from the data analysis.

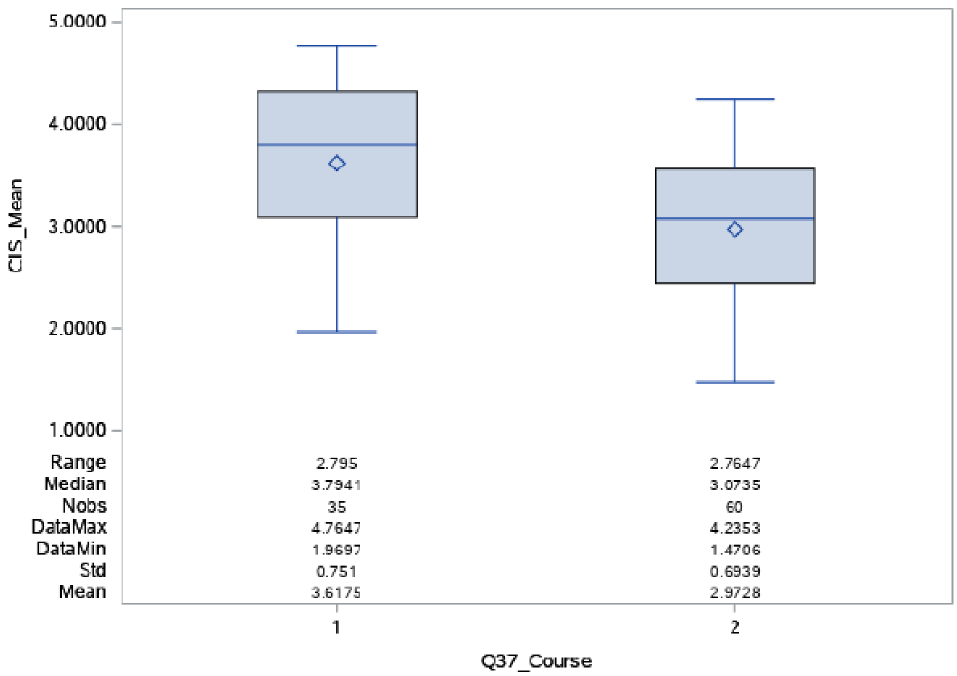


Figure E1. Distribution of overall motivation scores by course. 1 = treatment, 2 = control.

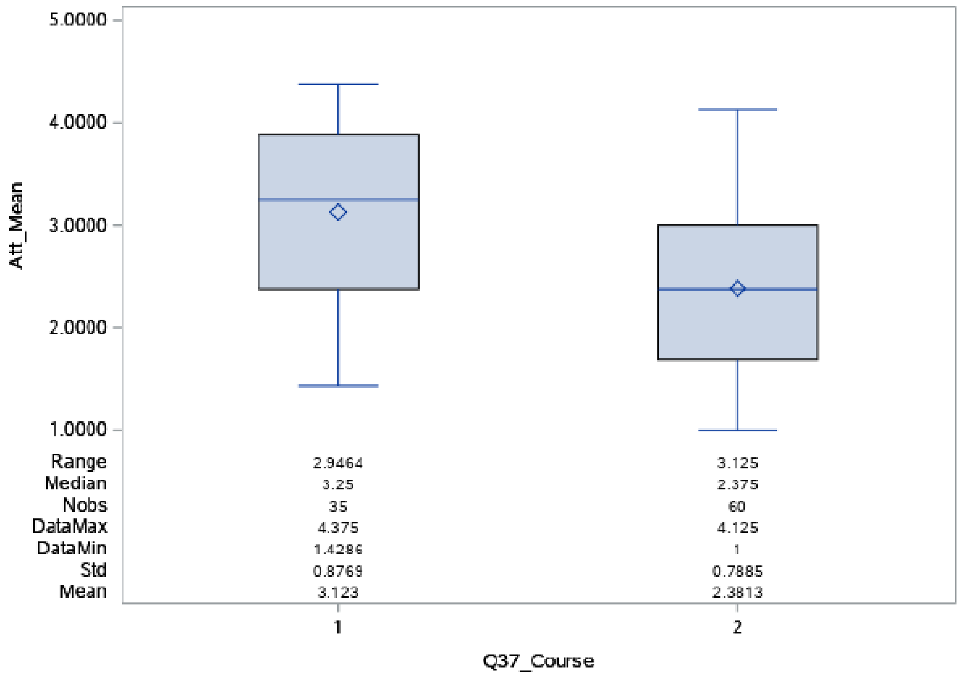


Figure E2. Distribution of attention scores by course. 1 = treatment, 2 = control.

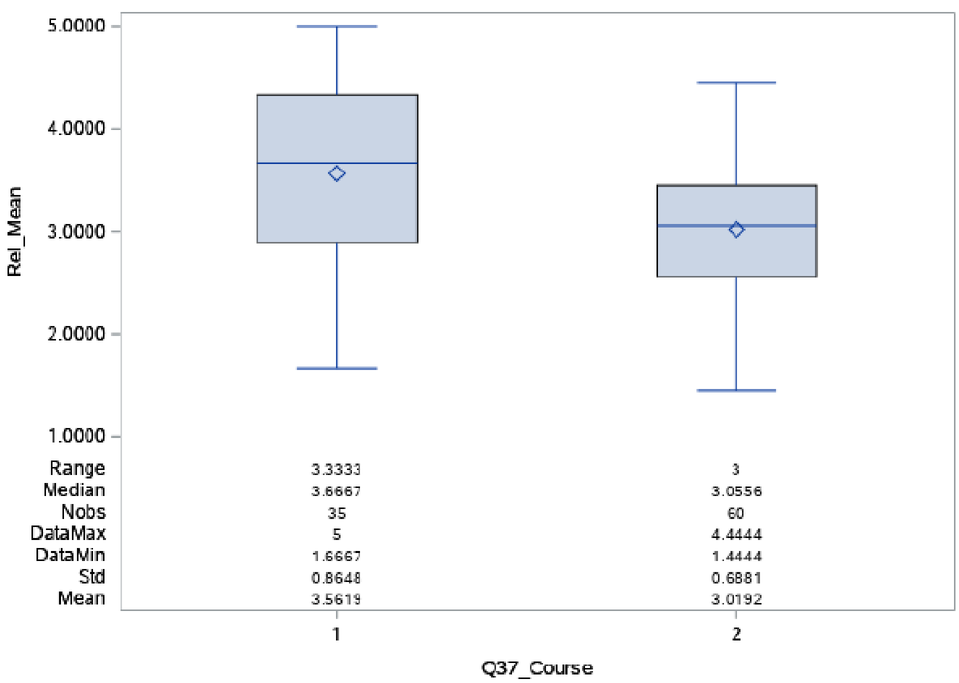


Figure E3. Distribution of relevance scores by course. 1 = treatment, 2 = control.

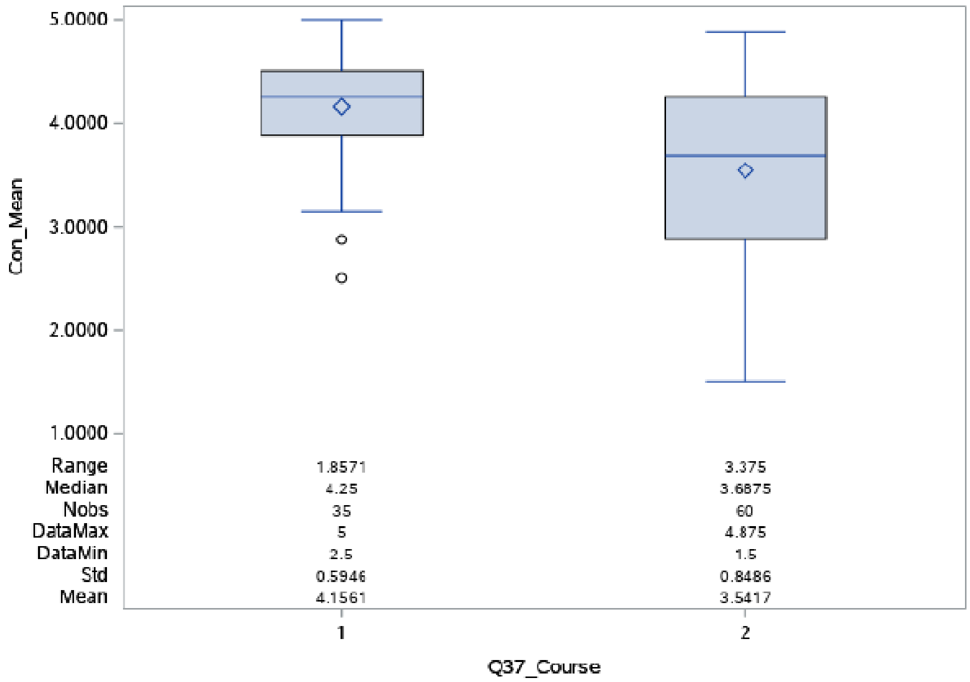


Figure E4. Distribution of confidence scores by course. 1 = treatment, 2 = control. Two outliers identified in treatment group.

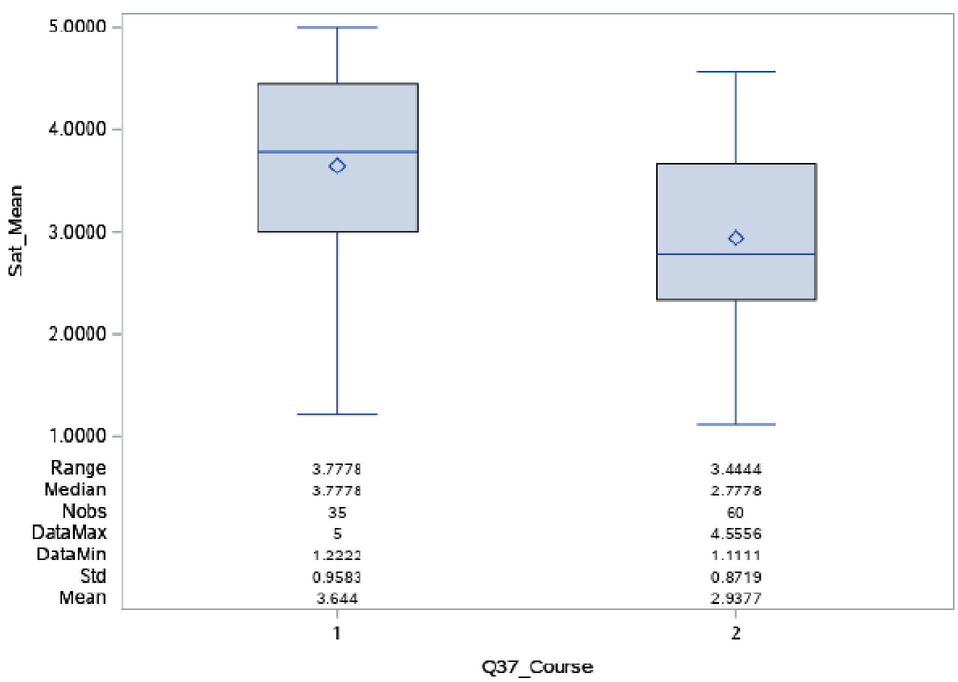


Figure E5. Distribution of satisfaction scores by course. 1 = treatment, 2 = control.

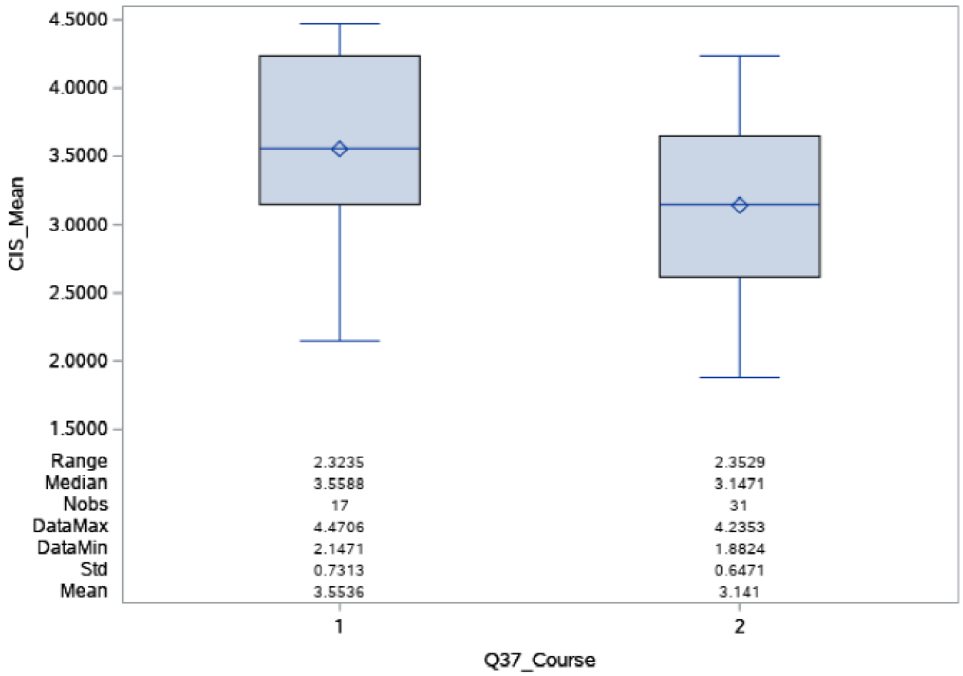


Figure E6. Distribution of overall motivation scores of males. 1 = treatment, 2 = control.

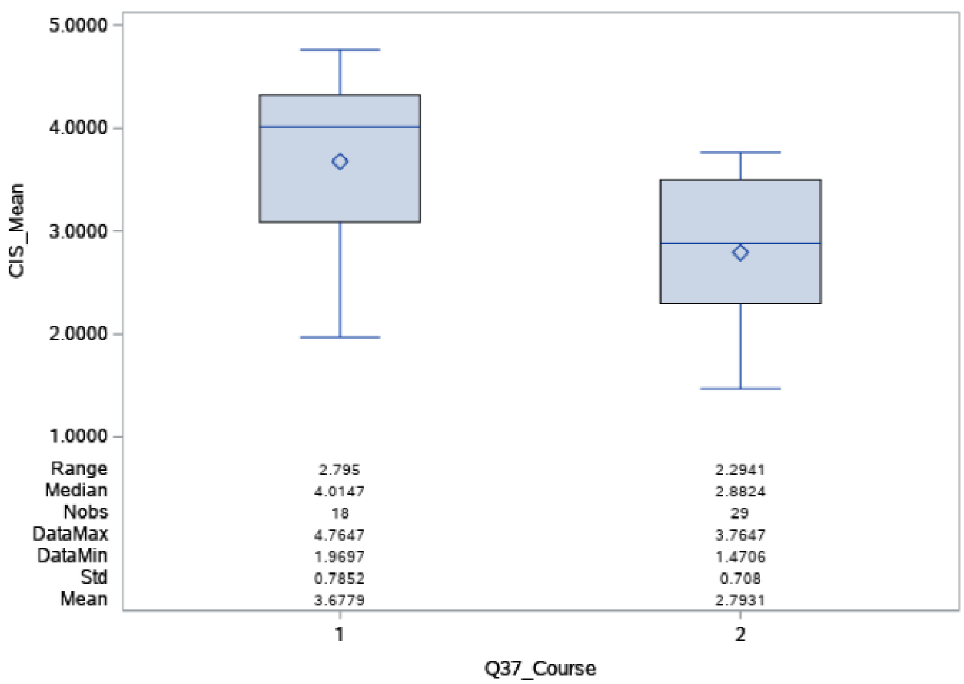


Figure E7. Distribution of overall motivation scores of females.

1 = treatment, 2 = control.

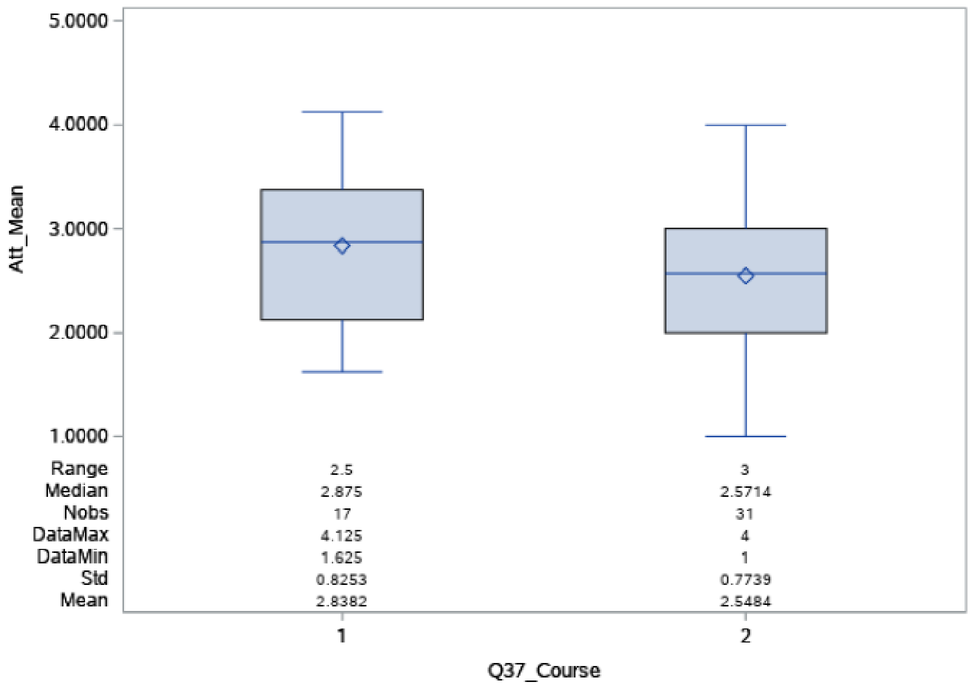


Figure E8. Distribution of attention scores of males. 1 = treatment, 2 = control.

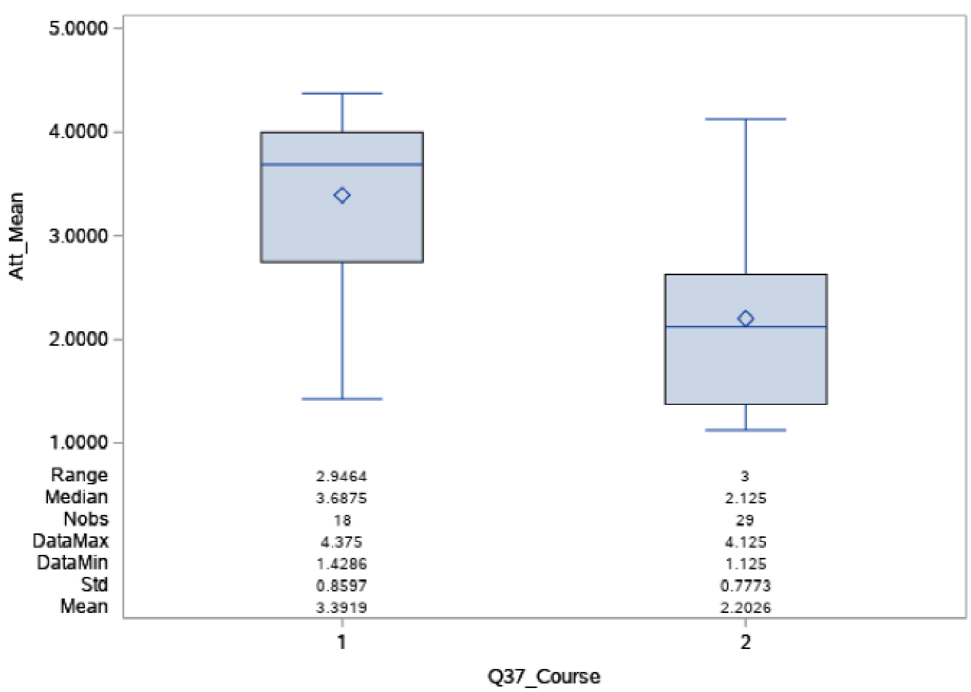


Figure E9. Distribution of attention scores of females. 1 = treatment, 2 = control.

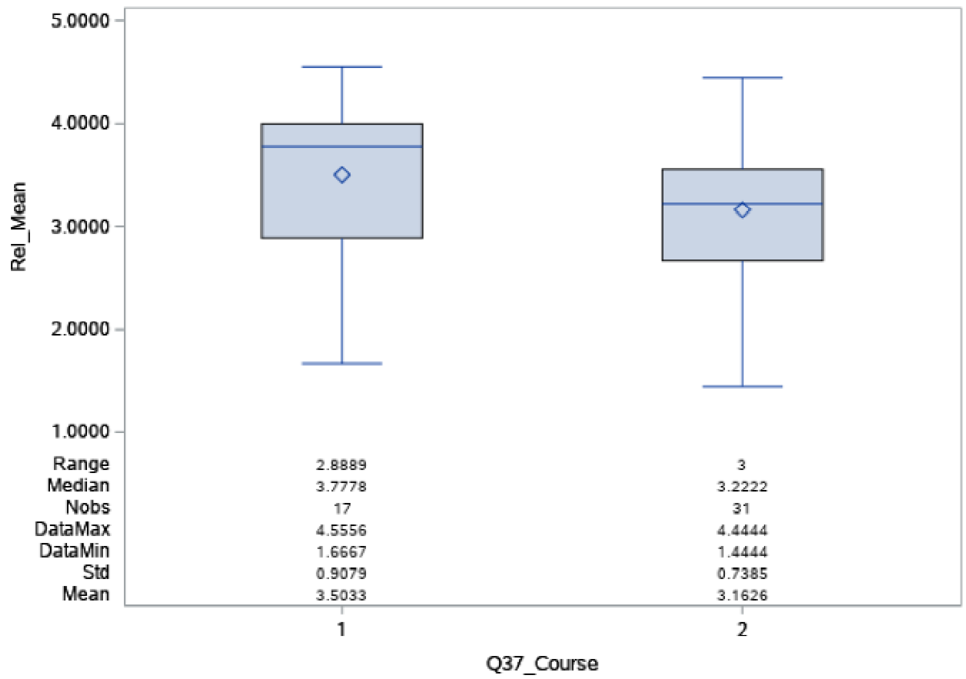


Figure E10. Distribution of relevance scores of males. 1 = treatment, 2 = control.

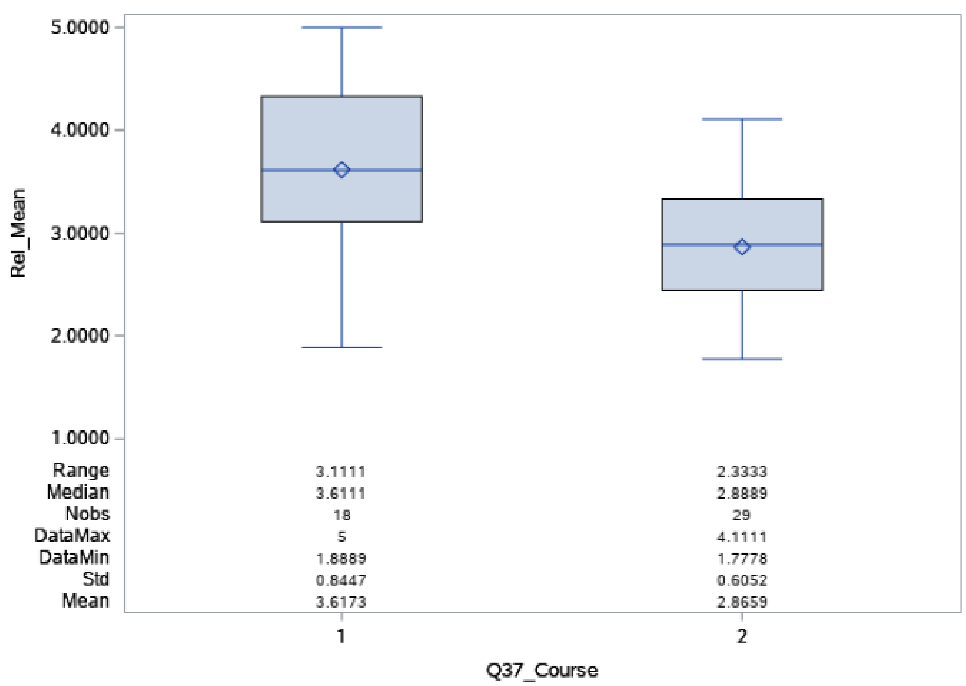


Figure E11. Distribution of relevance scores of females. 1 = treatment, 2 = control.

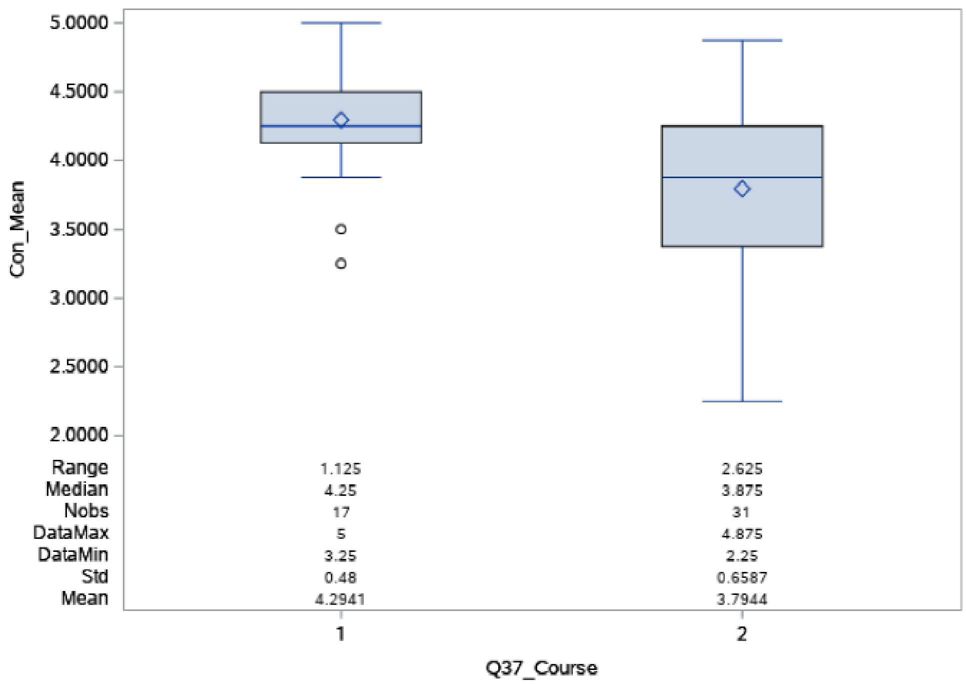


Figure E12. Distribution of confidence scores of males. 1 = treatment, 2 = control. Two outliers identified in treatment group.

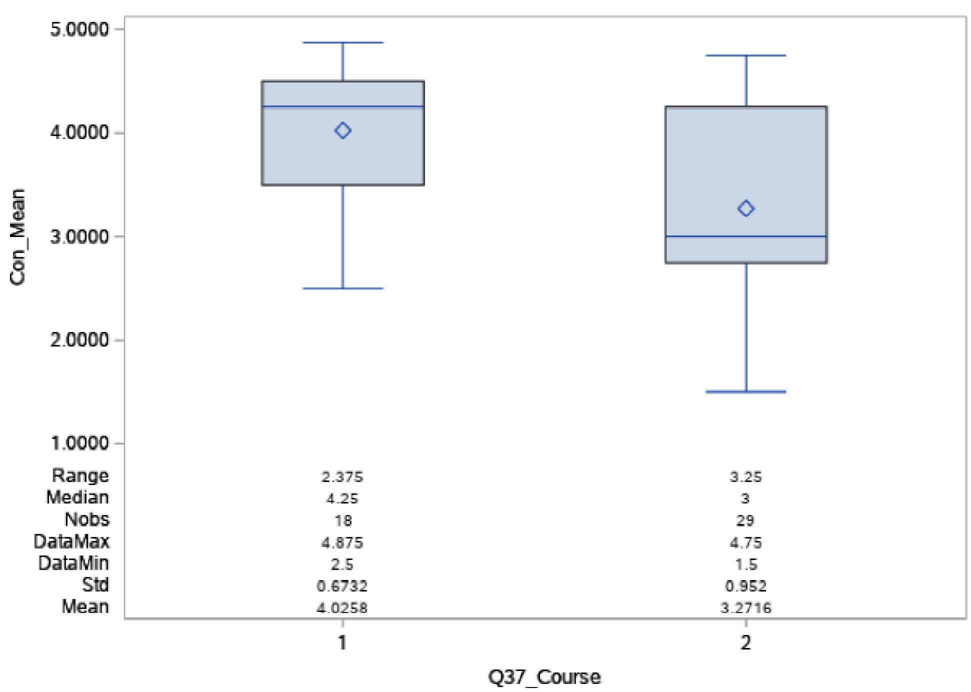


Figure E13. Distribution of confidence scores of females. 1 = treatment, 2 = control.

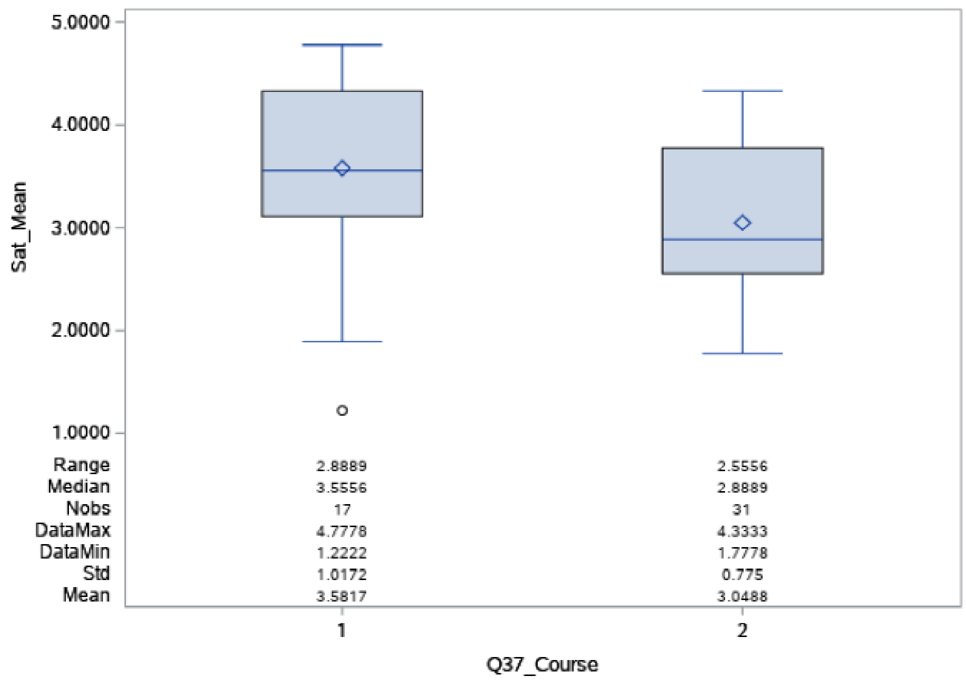


Figure E14. Distribution of satisfaction scores of males. 1 = treatment, 2 = control. One outlier identified in treatment group.

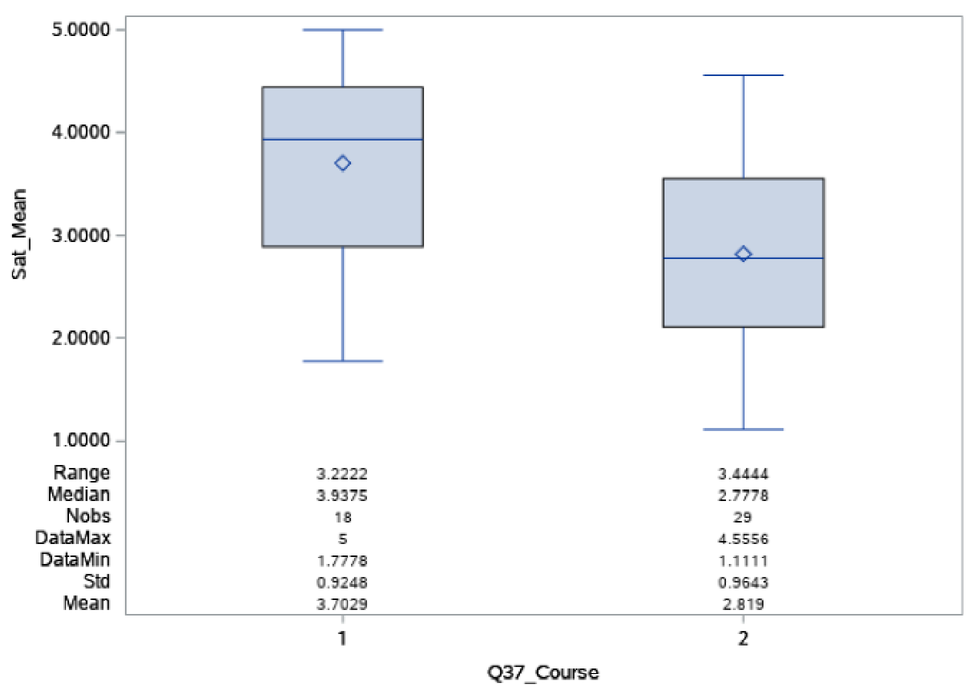


Figure E15. Distribution of satisfaction scores of females. 1 = treatment, 2 = control.

Assumption #5: The dependent variable should be approximately normally distributed.

A Kolmogorov-Smirnov (K-S) test was used to determine if data were normally distributed.

Table E1

K-S Test Results for Normal Distribution of Motivation Scores

Independent Variable Group	**Data Set**	**K-S Test p value**	**Conclusion**
treatment	overall motivation	0.040	data is not normally distributed
treatment	attention	0.150	data is normally distributed
treatment	relevance	0.150	data is normally distributed
treatment	confidence	0.010	data is not normally distributed
treatment	satisfaction	0.150	data is normally distributed
control	overall motivation	0.150	data is normally distributed
control	attention	0.150	data is normally distributed
control	relevance	0.150	data is normally distributed
control	confidence	0.010	data is not normally distributed
control	satisfaction	0.091	data is normally distributed
treatment males	overall motivation	0.150	data is normally distributed
treatment males	attention	0.150	data is normally distributed
treatment males	relevance	0.147	data is normally distributed
treatment males	confidence	0.150	data is normally distributed
treatment males	satisfaction	0.150	data is normally distributed
control males	overall motivation	0.150	data is normally distributed
control males	attention	0.150	data is normally distributed
control males	relevance	0.150	data is normally distributed
control males	confidence	0.150	data is normally distributed
control males	satisfaction	0.150	data is normally distributed
treatment females	overall motivation	0.026	data is not normally distributed
treatment females	attention	0.150	data is normally distributed
treatment females	relevance	0.150	data is normally distributed
treatment females	confidence	0.010	data is not normally distributed
treatment females	satisfaction	0.150	data is normally distributed
control females	overall motivation	0.150	data is normally distributed
control females	attention	0.150	data is normally distributed
control females	relevance	0.150	data is normally distributed
control females	confidence	0.045	data is not normally distributed
control females	satisfaction	0.150	data is normally distributed

Assumption #6: Variances are equal between groups.

A folded ***F*** test was used to test for equality of variance in motivation scores between all groups of the independent variable.

Table E2

Folded F Test Results for Equality of Variances in Motivation Scores

Independent Variable Groups Compared	Data Set	F Test p value	Conclusion
treatment and control	overall motivation	0.5845	variances are equal
treatment and control	attention	0.4668	variances are equal
treatment and control	relevance	0.1223	variances are equal
treatment and control	confidence	0.0275	variances are not equal
treatment and control	satisfaction	0.5160	variances are equal
treatment males and females	overall motivation	0.7803	variances are equal
treatment males and females	attention	0.8744	variances are equal
treatment males and females	relevance	0.7690	variances are equal
treatment males and females	confidence	0.1830	variances are equal
treatment males and females	satisfaction	0.6993	variances are equal
treatment males and control males	overall motivation	0.5460	variances are equal
treatment males and control males	attention	0.7359	variances are equal
treatment males and control males	relevance	0.3201	variances are equal
treatment males and control males	confidence	0.1829	variances are equal
treatment males and control males	satisfaction	0.1935	variances are equal
treatment females and control females	overall motivation	0.6099	variances are equal
treatment females and control females	attention	0.6191	variances are equal
treatment females and control females	relevance	0.1142	variances are equal
treatment females and control females	confidence	0.1378	variances are equal
treatment females and control females	satisfaction	0.8775	variances are equal

Appendix F

Statistical Analyses of Nonparametric Distributions of Motivation Data

For data sets of motivation scores that were not normally distributed, a Wilcoxon Rank-Sum Test was used to compare the outcomes between two groups. The results are shown below.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	Z	Pr > Z	Pr > |Z|	Pr > Z	Pr > |Z|
2172.500	3.7969	<.0001	0.0001	0.0001	0.0003
Z includes a continuity correction of 0.5.					

Ho = CIS_Mean scores of GIC and TG students are not significantly different from each other.
If p<.05, reject null.
Since p=.0001, reject null.
Conclusion: CIS_Mean scores of GIC and TG students ARE significantly different from each other.

Figure F1. Results of Wilcoxon Rank Sum Test comparing overall motivation scores of treatment and control groups.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	Z	Pr > Z	Pr > |Z|	Pr > Z	Pr > |Z|
2142.000	3.5700	0.0002	0.0004	0.0003	0.0006
Z includes a continuity correction of 0.5.					

Ho = Con_Mean scores of GIC and TG students are not significantly different from each other.
If p<.05, reject null.
Since p=.0004, reject null.
Conclusion: Con_Mean scores of GIC and TG students ARE significantly different from each other.

Figure F2. Results of Wilcoxon Rank Sum Test comparing confidence scores of treatment and control groups.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	Z	Pr < Z	Pr > |Z|	Pr < Z	Pr > |Z|
293.0000	-0.4129	0.3399	0.6797	0.3412	0.6823
Z includes a continuity correction of 0.5.					

Ho = CIS_Mean scores of GIC males and GIC females are not significantly different from each other.
If p<.05, reject null.
Since p=.6797, fail to reject null.
Conclusion: CIS_Mean scores of GIC males and GIC females are not significantly different from each other.

Figure F3. Results of Wilcoxon Rank Sum Test comparing overall motivation scores of males and females in the treatment group.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	Z	Pr > Z	Pr > |Z|	Pr > Z	Pr > |Z|
333.0000	0.8797	0.1895	0.3790	0.1926	0.3852
Z includes a continuity correction of 0.5.					

Ho = Con_Mean scores of GIC males and GIC females are not significantly different from each other.
If p<.05, reject null.
Since p=.3790, fail to reject null.
Conclusion: Con_Mean scores of GIC males and GIC females are not significantly different from each other.

Figure F4. Results of Wilcoxon Rank Sum Test comparing confidence scores of males and females in the treatment group.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	Z	Pr > Z	Pr > |Z|	Pr > Z	Pr > |Z|
591.0000	3.4705	0.0003	0.0005	0.0006	0.0011
Z includes a continuity correction of 0.5.					

Ho = CIS_Mean scores of GIC females and TG females are not significantly different from each other.
If p<.05, reject null.
Since p=.0005, reject null.
Conclusion: CIS_Mean scores of GIC females and TG females ARE significantly different from each other.

Figure F5. Results of Wilcoxon Rank Sum Test comparing overall motivation scores of females in the treatment group and females in the control group.

Wilcoxon Two-Sample Test					
				t Approximation	
Statistic	Z	Pr > Z	Pr > |Z|	Pr > Z	Pr > |Z|
554.5000	2.6802	0.0037	0.0074	0.0051	0.0102
Z includes a continuity correction of 0.5.					

Ho = Con_Mean scores of GIC females and TG females are not significantly different from each other.
If p<.05, reject null.
Since p=.0074, reject null.
Conclusion: Con_Mean scores of GIC females and TG females ARE significantly different from each other.

Figure F6. Results of Wilcoxon Rank Sum Test comparing confidence scores of females in the treatment group and females in the control group.